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Weakly $g^{\star}$-closed Sets

Research Article

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Abstract: Veera kumar [14] introduced the class of $g^{\star}$-closed sets. We introduce a new class of generalized closed sets called weakly $g^{\star}$-closed sets which contains the above mentioned class. Also, we investigate the relationships among the related generalized closed sets.

Keywords: $g^{\star}$-closed set, w$g^{\star}$-closed set, $g^{\star}$-continuity, w$g^{\star}$-continuity, contra $g^{\star}$-continuity.

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1. Introduction

Veerakumar [14] studied and investigated properties of the notion of $g^{\star}$-closed sets. In this paper, we introduce a new class of generalized closed sets called weakly $g^{\star}$-closed sets which contains the above mentioned class. Also, we investigate the relationships among the related generalized closed sets.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (briefly X and Y) represent non-empty topological spaces. For a subset A of a space X, cl(A), int(A) and C(A) denotes the closure of A, the interior of A and the complement of A respectively.
Recall that a subset A of a space X is called nowhere dense if int(cl(A)) = $\emptyset$.

Definition 2.1. *Let A be a subset of a space X. Then A is called*

(1). an α-open [9] if $A \subseteq$ int(cl(int(A))).

(2). an α-closed if C(A) is an α-open.

(3). semi-open [6] if $A \subseteq$ cl(int(A)).

(4). semi-closed if C(A) is semi-open.

(5). regular open [11] if A = int(cl(A)).

(6). regular closed if C(A) is regular open.

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The α-closure of a subset A of X, denoted by $\alpha cl(A)$, is defined as the intersection of all α-closed sets containing A.

Definition 2.2. *Let X and Y be two topological spaces. A function $f : X \to Y$ is called perfectly continuous [1] (resp. R-map [2]) if $f^{-1}(V)$ is clopen (resp. regular open) in X for regular open set V of Y.*

Definition 2.3. *A subset A of a space X is called π-open [3] if the finite union of regular open sets.*

Definition 2.4. *A subset A of a topological space X is called*

(1). a weakly g-closed (briefly, wg-closed) set [12] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(2). a weakly πg-closed (briefly, $w\pi g$-closed) set [10] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is π-open in X.

(3). a regular weakly generalized closed (briefly, rwg-closed) set [8] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Definition 2.5. *A topological space X is said to be almost connected [4] if X cannot be written as a disjoint union of two non-empty regular open sets.*

Remark 2.6 ([3]). *For a subset of a topological space, we have following implications:*

$$\textbf{regular open} \Rightarrow \pi\textbf{-open} \Rightarrow \textbf{open}$$

Definition 2.7. *Let A be a subset of a space X. Then A is called*

(1). g-closed [5] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X;

(2). g-closed if $C(A)$ is g-open.

(3). $g^\star$-closed [14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X;

(4). $g^\star$-closed if $C(A)$ is $g^\star$-open.

(5). αg-closed [7] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X;

(6). αg-closed if $C(A)$ is αg-open.

Remark 2.8. *In a space X,*

(1). every closed set is $g^\star$-closed but not conversely.[14]

(2). every closed set is g-closed but not conversely.[5]

3. Weakly $g^\star$-closed Sets

We introduce the definition of weakly $g^\star$-closed sets in topological spaces and study the relationships of such sets.

Definition 3.1. *A subset A of a topological space X is called a weakly $g^\star$-closed (briefly, $wg^\star$-closed) set if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.*

Theorem 3.2. *Every $g^\star$-closed set is $wg^\star$-closed but not conversely.*

Example 3.3. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$. Then the set $\{a\}$ is $wg^\star$-closed set but not a $g^\star$-closed in X.*

Theorem 3.4. *Every $wg^\star$-closed set is wg-closed but not conversely.*

Proof. Let A be any w$g^{\star}$-closed set and U be any open set containing A. Then U is an g-open set containing A. We have cl(int(A)) $\subseteq$ U. Thus, A is wg-closed. $\square$

Example 3.5. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$. Then the set $\{a, b\}$ is wg-closed but not a wg*-closed.*

Theorem 3.6. *Every wg*-closed set is wπg-closed but not conversely.*

Proof. Let A be any w$g^{\star}$-closed set and U be any π-open set containing A. Then U is an g-open set containing A. We have cl(int(A)) $\subseteq$ U. Thus, A is wπg-closed. $\square$

Example 3.7. *In Example 3.5, the set $\{a, c\}$ is wπg-closed set but not a wg*-closed.*

Theorem 3.8. *Every wg*-closed set is rwg-closed but not conversely.*

Proof. Let A be any w$g^{\star}$-closed set and U be any regular open set containing A. Then U is an g-open set containing A. We have cl(int(A)) $\subseteq$ U. Thus, A is rw$g$-closed. $\square$

Example 3.9. *In Example 3.5, the set $\{a\}$ is rwg-closed set but not a wg*-closed.*

Theorem 3.10. *If a subset A of a topological space X is both closed and g-closed, then it is wg*-closed in X.*

Proof. Let A be a g-closed set in X and U be any open set containing A. Then U $\supseteq$ cl(A) $\supseteq$ cl(int(cl(A))). Since A is closed, U $\supseteq$ cl(int(A)) and U is g-open set containing A. Hence A is w$g^{\star}$-closed in X. $\square$

Theorem 3.11. *If a subset A of a topological space X is both open and wg*-closed, then it is closed.*

Proof. Since A is both open and w$g^{\star}$-closed, A $\supseteq$ cl(int(A)) = cl(A) and hence A is closed in X. $\square$

Corollary 3.12. *If a subset A of a topological space X is both open and wg*-closed, then it is both regular open and regular closed in X.*

Theorem 3.13. *Let X be a topological space and A $\subseteq$ X be open. Then, A is wg*-closed if and only if A is g*-closed.*

Proof. Let A be $g^{\star}$-closed. By Theorem 3.2, it is w$g^{\star}$-closed. Conversely, let A be w$g^{\star}$-closed. Since A is open, by Theorem 3.11, A is closed. Hence A is $g^{\star}$-closed. $\square$

Theorem 3.14. *If a set A of X is wg*-closed, then cl(int(A)) $-$ A contains no non-empty g-closed set.*

Proof. Let F be an g-closed set such that F $\subseteq$ cl(int(A)) $-$ A. Since F^c is g-open and A $\subseteq$ F^c, from the definition of w$g^{\star}$-closedness it follows that cl(int(A)) $\subseteq$ F^c. i.e., F $\subseteq$ (cl(int(A)))^c. This implies that F $\subseteq$ (cl(int(A))) $\cap$ (cl(int(A)))^c = ϕ. $\square$

Theorem 3.15. *If a subset A of a topological space X is nowhere dense, then it is wg*-closed.*

Proof. Since int(A) $\subseteq$ int(cl(A)) and A is nowhere dense, int(A) = ϕ. Therefore cl(int(A)) = ϕ and hence A is w$g^{\star}$-closed in X. The converse of Theorem 3.15 need not be true as seen in the following example. $\square$

Example 3.16. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. Then the set $\{a\}$ is wg*-closed set but not nowhere dense in X.*

Remark 3.17. *The following Examples show that wg*-closedness and semi-closedness are independent.*

Example 3.18. *In Example 3.3, the set $\{a, c\}$ is wg*-closed but not semi-closed in X.*

Example 3.19. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then the set $\{a\}$ is semi-closed but not $wg^\star$-closed in X.*

Remark 3.20. *From the above discussions and known results, we obtain the following diagram, where $A \to B$ represents A implies B but not conversely.*

Diagram

$$\textbf{closed} \Rightarrow \boldsymbol{wg^\star}\textbf{-closed} \Rightarrow \boldsymbol{wg}\textbf{-closed} \Rightarrow \boldsymbol{w\pi g}\textbf{-closed} \Rightarrow \boldsymbol{rwg}\textbf{-closed}$$

Definition 3.21. *A subset A of a topological space X is called $wg^\star$-open set if C(A) is $wg^\star$-closed in X.*

Proposition 3.22. *Every g-open set is $wg^\star$-open but not conversely.*

Theorem 3.23. *A subset A of a topological space X is $wg^\star$-open if $G \subseteq int(cl(A))$ whenever $G \subseteq A$ and G is g-closed.*

Proof. Let A be any $wg^\star$-open. Then A^c is $wg^\star$-closed. Let G be an g-closed set contained in A. Then G^c is an g-open set containing A^c. Since A^c is $wg^\star$-closed, we have $cl(int(A^c)) \subseteq G^c$. Therefore $G \subseteq int(cl(A))$.
Conversely, we suppose that $G \subseteq int(cl(A))$ whenever $G \subseteq A$ and G is g-closed. Then G^c is an g-open set containing A^c and $G^c \supseteq (int(cl(A)))^c$. It follows that $G^c \supseteq cl(int(A^c))$. Hence A^c is $wg^\star$-closed and so A is $wg^\star$-open. ☐

4. Weakly $g^\star$-continuous Functions

Definition 4.1. *Let X and Y be two topological spaces. A function $f: X \to Y$ is called*

(1). weakly $g^\star$-continuous (briefly, $wg^\star$-continuous) if $f^{-1}(U)$ is a $wg^\star$-open set in X for each open set U of Y.

(2). $g^\star$-continuous [14] if $f^{-1}(U)$ is a $g^\star$-open set in X for each open set U of Y.

(3). contra $g^\star$-continuous [13] if $f^{-1}(V)$ is $g^\star$-closed set of X for every open set V of Y.

Example 4.2. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. The function $f: X \to Y$ defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$ is $wg^\star$-continuous, because every open subset of Y is $wg^\star$-closed in X.*

Theorem 4.3. *Every $g^\star$-continuous function is $wg^\star$-continuous.*

Proof. It follows from Theorem 3.2. ☐

The converse of Theorem 4.3 need not be true as seen in the following example.

Example 4.4. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{b\}, Y\}$. Let $f: X \to Y$ be the identity function. Then f is $wg^\star$-continuous but not $g^\star$-continuous.*

Theorem 4.5. *A function $f: X \to Y$ is $wg^\star$-continuous if and only if $f^{-1}(U)$ is a $wg^\star$-closed set in X for each closed set U of Y.*

Proof. Let U be any closed set of Y. According to the assumption $f^{-1}(U^c) = X \setminus f^{-1}(U)$ is $wg^\star$-open in X, so $f^{-1}(U)$ is $wg^\star$-closed in X.
The converse can be proved in a similar manner. ☐

Definition 4.6. *A topological space X is said to be locally $g^\star$-indiscrete if every $g^\star$-open set of X is closed in X.*

Theorem 4.7. *Let f : $X \to Y$ be a function. If f is contra $g^\star$-continuous and X is locally $g^\star$-indiscrete, then f is continuous.*

Proof. Let V be a closed in Y. Since f is contra $g^\star$-continuous, $f^{-1}(V)$ is $g^\star$-open in X. Since X is locally $g^\star$-indiscrete, $f^{-1}(V)$ is closed in X. Hence f is continuous. □

Theorem 4.8. *Let $f : X \to Y$ be a function. If f is contra $g^\star$-continuous and X is locally $g^\star$-indiscrete, then f is w$g^\star$-continuous.*

Proof. Let f : X → Y be contra $g^\star$-continuous and X is locally $g^\star$-indiscrete. By Theorem 4.7, f is continuous, then f is w$g^\star$-continuous. □

Proposition 4.9. *If $f : X \to Y$ is perfectly continuous and w$g^\star$-continuous, then it is R-map.*

Proof. Let V be any regular open subset of Y. According to the assumption, $f^{-1}(V)$ is both open and closed in X. Since $f^{-1}(V)$ is closed, it is w$g^\star$-closed. We have $f^{-1}(V)$ is both open and w$g^\star$-closed. Hence, by Corollary 3.12, it is regular open in X, so f is R-map. □

Definition 4.10. *A topological space X is called $g^\star$-compact if every cover of X by $g^\star$-open sets has a finite subcover.*

Definition 4.11. *A topological space X is weakly $g^\star$-compact (briefly, w$g^\star$-compact) if every w$g^\star$-open cover of X has a finite subcover.*

Remark 4.12. *Every w$g^\star$-compact space is $g^\star$-compact.*

Theorem 4.13. *Let $f : X \to Y$ be surjective w$g^\star$-continuous function. If X is w$g^\star$-compact, then Y is compact.*

Proof. Let $\{A_i : i \in I\}$ be an open cover of Y. Then $\{ f^{-1}(A_i) : i \in I\}$ is a w$g^\star$-open cover in X. Since X is w$g^\star$-compact, it has a finite subcover, say $\{ f^{-1}(A_1), f^{-1}(A_2),...., f^{-1}(A_n)\}$. Since f is surjective $\{A_1, A_2,...., A_n\}$ is a finite subcover of Y and hence Y is compact. □

Definition 4.14. *A topological space X is called*

(1). weakly $g^\star$-connected (briefly, w$g^\star$-connected) if X cannot be written as the disjoint union of two non-empty w$g^\star$-open sets.

(2). $g^\star$-connected [13] if X cannot be written as the disjoint union of two non-empty $g^\star$-open sets.

Theorem 4.15. *If a topological space X is w$g^\star$-connected, then X is almost connected (resp. $g^\star$-connected).*

Proof. It follows from the fact that each regular open set (resp. $g^\star$-open set) is w$g^\star$-open. □

Theorem 4.16. *For a topological space X, the following statements are equivalent:*

(1). X is w$g^\star$-connected.

(2). The empty set ϕ and X are only subsets which are both w$g^\star$-open and w$g^\star$-closed.

(3). Each w$g^\star$-continuous function from X into a discrete space Y which has at least two points is a constant function.

Proof. (1) $\Rightarrow$ (2). Let $S \subseteq X$ be any proper subset, which is both w$g^\star$-open and w$g^\star$-closed. Its complement $X \setminus S$ is also w$g^\star$-open and w$g^\star$-closed. Then $X = S \cup (X \setminus S)$ is a disjoint union of two non-empty w$g^\star$-open sets which is a contradiction with the fact that X is w$g^\star$-connected. Hence, $S = \phi$ or X.

(2) $\Rightarrow$ (1). Let $X = A \cup B$ where $A \cap B = \phi$, $A \neq \phi$, $B \neq \phi$ and A, B are w$g^\star$-open. Since $A = X \setminus B$, A is w$g^\star$-closed. According to the assumption $A = \phi$, which is a contradiction.

(2) $\Rightarrow$ (3). Let $f : X \to Y$ be a w$g^\star$-continuous function where Y is a discrete space with at least two points. Then $f^{-1}(\{y\})$ is w$g^\star$-closed and w$g^\star$-open for each $y \in Y$ and $X = \cup\ \{ f^{-1}(\{y\}) : y \in Y\}$. According to the assumption, $f^{-1}(\{y\}) = \phi$ or $f^{-1}(\{y\}) = X$. If $f^{-1}(\{y\}) = \phi$ for all $y \in Y$, f will not be a function. Also there is no exist more than one $y \in Y$ such that $f^{-1}(\{y\}) = X$. Hence, there exists only one $y \in Y$ such that $f^{-1}(\{y\}) = X$ and $f^{-1}(\{y_1\}) = \phi$ where $y \neq y_1 \in Y$. This shows that f is a constant function.

(3) $\Rightarrow$ (2). Let $S \neq \phi$ be both w$g^\star$-open and w$g^\star$-closed in X. Let $f : X \to Y$ be a w$g^\star$-continuous function defined by $f(S) = \{a\}$ and $f(X \setminus S) = \{b\}$ where $a \neq b$. Since f is constant function we get $S = X$. □

Theorem 4.17. *Let $f : X \to Y$ be a w$g^\star$-continuous surjective function. If X is w$g^\star$-connected, then Y is connected.*

Proof. We suppose that Y is not connected. Then $Y = A \cup B$ where $A \cap B = \phi$, $A \neq \phi$, $B \neq \phi$ and A, B are open sets in Y. Since f is w$g^\star$-continuous surjective function, $X = f^{-1}(A) \cup f^{-1}(B)$ are disjoint union of two non-empty w$g^\star$-open subsets. This is contradiction with the fact that X is w$g^\star$-connected. □

5. Weakly $g^\star$-open and Weakly $g^\star$-closed Functions

Definition 5.1. *Let X and Y be topological spaces. A function $f : X \to Y$ is called*

(1). weakly $g^\star$-open (briefly, w$g^\star$-open) if f(V) is a w$g^\star$-open set in Y for each open set V of X.

(2). $g^\star$-open [13] if f(V) is a $g^\star$-open set in Y for each open set V of X.

Remark 5.2. *Every $g^\star$-open function is w$g^\star$-open but not conversely.*

Example 5.3. *Let $X = Y = \{a, b, c, d\}$ with $\tau = \{\phi, \{a\}, \{a, b, d\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, Y\}$. Let $f : X \to Y$ be the identity function. Then f is w$g^\star$-open but not $g^\star$-open.*

Definition 5.4. *Let X and Y be topological spaces. A function $f : X \to Y$ is called weakly $g^\star$-closed (briefly, w$g^\star$-closed) if f(V) is a w$g^\star$-closed set in Y for each closed set V of X. It is clear that an open function is w$g^\star$-open and a closed function is w$g^\star$-closed.*

Theorem 5.5. *Let X and Y be topological spaces. A function $f : X \to Y$ is w$g^\star$-closed if and only if for each subset B of Y and for each open set G containing $f^{-1}(B)$ there exists a w$g^\star$-open set F of Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq G$.*

Proof. Let B be any subset of Y and let G be an open subset of X such that $f^{-1}(B) \subseteq G$. Then $F = Y \setminus f(X \setminus G)$ is w$g^\star$-open set containing B and $f^{-1}(F) \subseteq G$.

Conversely, let U be any closed subset of X. Then $f^{-1}(Y \setminus f(U)) \subseteq X \setminus U$ and $X \setminus U$ is open. According to the assumption, there exists a w$g^\star$-open set F of Y such that $Y \setminus f(U) \subseteq F$ and $f^{-1}(F) \subseteq X \setminus U$. Then $U \subseteq X \setminus f^{-1}(F)$. From $Y \setminus F \subseteq f(U) \subseteq f(X \setminus f^{-1}(F)) \subseteq Y \setminus F$ it follows that $f(U) = Y \setminus F$, so f(U) is w$g^\star$-closed in Y. Therefore f is a w$g^\star$-closed function. □

Remark 5.6. *The composition of two w$g^\star$-closed functions need not be a w$g^\star$-closed as it can be seen from the following Example.*

Example 5.7. *Let $X = Y = Z = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{a, b\}, X\}$, $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$ and $\eta = \{\phi, \{a, b\}, Z\}$. We define $f : X \to Y$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$ and let $g : Y \to Z$ be the identity function. Hence both f and g are $wg^\star$-closed functions. For a closed set $U = \{b, c\}$ in X, $(g\ o\ f)(U) = g(f(U)) = g(\{a, b\}) = \{a, b\}$ which is not $wg^\star$-closed in Z. Hence the composition of two $wg^\star$-closed functions need not be a $wg^\star$-closed.*

Theorem 5.8. *Let X, Y and Z be topological spaces. If $f : X \to Y$ is a closed function and $g : Y \to Z$ is a $wg^\star$-closed function, then $g \circ f : X \to Z$ is a $wg^\star$-closed function.*

Definition 5.9. *A function $f : X \to Y$ is called*

(1). a weakly $g^\star$-irresolute (briefly, $wg^\star$-irresolute) if $f^{-1}(U)$ is a $wg^\star$-open set in X for each $wg^\star$-open set U of Y.

(2). $g^\star$-irresolute [14] if $f^{-1}(U)$ is a $g^\star$-open set in X for each $g^\star$-open set U of Y.

(3). an αg-irresolute [7] if $f^{-1}(U)$ is an αg-open set in X for each αg-open set U of Y.

Example 5.10. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{b\}, Y\}$. Let $f : X \to Y$ be the identity function. Then f is $wg^\star$-irresolute.*

Remark 5.11. *The following Examples show that αg-irresoluteness and $wg^\star$-irresoluteness are independent of each other.*

Example 5.12. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Let $f : X \to Y$ be the identity function. Then f is $wg^\star$-irresolute but not αg-irresolute.*

Example 5.13. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f : X \to Y$ be the identity function. Then f is αg-irresolute but not $wg^\star$-irresolute.*

Remark 5.14. *Every $g^\star$-irresolute function is $wg^\star$-continuous but not conversely. Also, the concepts of $g^\star$-irresoluteness and $wg^\star$-irresoluteness are independent of each other.*

Example 5.15. *Let $X = Y = \{a, b, c, d\}$ with $\tau = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b, d\}, Y\}$. Let $f : X \to Y$ be the identity function. Then f is $wg^\star$-continuous but not $g^\star$-irresolute.*

Example 5.16. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Let $f : X \to Y$ be the identity function. Then f is $wg^\star$-irresolute but not $g^\star$-irresolute.*

Example 5.17. *In Example 5.13, f is $g^\star$-irresolute but not $wg^\star$-irresolute.*

Theorem 5.18. *The composition of two $wg^\star$-irresolute functions is also $wg^\star$-irresolute.*

Theorem 5.19. *Let $f : X \to Y$ and $g : Y \to Z$ be functions such that $g \circ f : X \to Z$ is $wg^\star$-closed function. Then the following statements hold:*

(1). if f is continuous and injective, then g is $wg^\star$-closed.

(2). if g is $wg^\star$-irresolute and surjective, then f is $wg^\star$-closed.

Proof.

(1). Let F be a closed set of Y. Since $f^{-1}(F)$ is closed in X, we can conclude that $(g \circ f)(f^{-1}(F))$ is $wg^\star$-closed in Z. Hence g(F) is $wg^\star$-closed in Z. Thus g is a $wg^\star$-closed function.

(2). It can be proved in a similar manner as (1).

□

Theorem 5.20. *If $f : X \to Y$ is an $wg^{\star}$-irresolute function, then it is $wg^{\star}$-continuous.*

Remark 5.21. *The converse of the above Theorem need not be true in general. The function $f : X \to Y$ in the Example 5.13 is $wg^{\star}$-continuous but not $wg^{\star}$-irresolute.*

Theorem 5.22. *If $f : X \to Y$ is surjective $wg^{\star}$-irresolute function and X is $wg^{\star}$-compact, then Y is $wg^{\star}$-compact.*

Theorem 5.23. *If $f : X \to Y$ is surjective $wg^{\star}$-irresolute function and X is $wg^{\star}$-connected, then Y is $wg^{\star}$-connected.*

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