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Minimizing the Range of Lateness for n-Jobs, 3-Machines Sequencing Problem with Due Date, Transportation Times and Equivalent Job for Block of Jobs – A Heuristic Approach

Research Article

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Abstract: Here a heuristic approached has been developed to minimize the range of lateness in sequencing problem of n-jobs, 3-machine with due-date and equivalent job for block of jobs. The approach is based on three theorems. This paper investigates some conditions to be imposed on times transportation and processing times. Minimization of range of lateness is the criterion to find optimal sequence of jobs. A rule to determine equivalent job and its processing times for jobs in block has also been determined here. The work has been supported by a numerical example.

Keywords: Job blocks, Due-dates, Sequencing, Range of lateness, Optimization.

1. Introduction

The problem of n-jobs, on a single machine with due date was studied by many researchers with different objectives of finding optimal sequence including minimization of flow time, total lateness, waiting time variance of flow time etc. Gupta and Sen [7] studied the problem of n-jobs on a single machine with due date the objective of minimizing the difference between maximum and minimum job lateness or the range of lateness. Gupta and sen [7] justified that such problem are important in real life whenever it is desirable to give equal treatment to all customers (jobs).

Let t_i = processing time, c_i = completion time and d_i = due date of i^{th} job, $i = 1, 2, \ldots, n$. Lateness of i^{th} job, $L_i = c_i d_i$, $i = 1, 2, \ldots, n$. Slack time of i^{th} job = $d_i t_i'$. For a given sequence s, the value of objective function is:

$$z(s) = \max\{Li\} - \min\{Li\},\ i = 1, 2, \ldots, n.$$

The optimal sequence defined by term is that which minimizes L(s) or the sequence which minimize range of lateness is optimal sequence. Gupta and Sen [5] proved in their paper that a sequence in which jobs are arranged as MST (Minimum

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Slack Time) rule, is optimal sequence, if jobs are also in EDD (Early Due Date) order. That is, the sequence in which jobs are in both MST and EDD minimizes the range of lateness. The concept of minimization of range of lateness in n-job, one machine with due date due to Gupta & Sen was extended to n-jobs, 2-machine sequencing problem Ikram and Tahir [4]. They studied n-jobs, 2-machine problem with due date satisfying some conditions imposed on processing times of jobs on two machines A and B in order AB with the objective to minimize the range of lateness. They denoted A_i, B_i as processing times of jobs i, $i = 1, 2, \ldots, n$ on machines A and B respectively.

$$d_i = \text{due date of } i^{th} \text{ job}, i = 1, 2, \ldots, n.$$

$$c_i = \text{completion time of job i on machine C.}$$

Lateness of job i, $L_i = c_i d_i$ and slack time of job $i = d_i(B_i)$.

Ikram and Tahir [4] proved that if $\max\{Ai\} \leq \min\{Bi\}$, then a sequence in which jobs are arranged according to MST rule, minimizes range of lateness if jobs are also in EDD order in that sequence. Ikram and Tahir [3], introduced the concept of transportation time due to Maggu and Das [5] in the problem studied by Ikram and Tahir [4]. Further Ikram and Singh [5], introduced the concept of equivalent job for jobs in block due to Maggu and Das [3], in the problem of Ikram and Tahir [4]. Here both the concept of transportation time and equivalent job for block of jobs have been introduced simultaneously with the objective to minimize range of lateness in n-job, 3-machines sequencing problem, satisfying some conditions imposed on transportation times and processing times.

2. Formulation of Problem

Table 1. **General Tabular Presentation of Sequencing Problem**

	Machine A (A_i)	Transportation Time (t_i)	Machine B (B_i)	Transportation Time (g_i)	Machine C (C_i)	Due date (d_i)
1	A_1	t_1	B_1	g_1	C_1	d_1
2	A_2	t_2	B_2	g_2	C_2	d_2
3	A_3	t_3	B_3	g_3	C_3	d_3
4	N	...	...	...	...	...
5	N	...	...	...	...	...
N	N	...	...	...	...	...
n	A_n	t_n	B_n	g_n	C_n	d_n

Where A_i, B_i and C_i are processing time of job i, $i = 1, 2, \ldots, n$ on machine A , B and C respectively; t_i, g_i and d_i are transportation time and due date of job i, $i = 1, 2, \ldots, n$. three jobs are forming a block. The problem is to determine an optimal sequence which minimizes the range of lateness. Let c_i = completion time of job i, on machine C. Lateness of job i, $L_i = c_i - d_i$. Slack time of job $i = d_i - C_i$. The value of objective function for a given sequence s is $z(s) = \max\{Li\} - \min\{Li\}$.

2.1. Suggested Conditions

$$\max\{A_i + t_i + B_i + g_i\} \leq \min\{C_i\} \tag{1}$$

Theorem 2.1. *Let $[1], [2], [3], \ldots, [i-1], [i], \ldots, [n]$ be a sequence of jobs where i denotes the job which is sequenced at i^{th} place. Then the completion time of i^{th} job on machine C in the above stated problem satisfying condition given in (1), is given by Completion time* $= A_1 + t_1 + B_1 + g_1 + \sum_{j=1}^{[i]} C_j$

Proof. In the sequence $[1], [2], [3], \ldots, [i-1], [i], \ldots, [n]$, the completion time of 1^{st} job on machine $C = A_1 + t_1 + B_1 + g_1$. Completion time of 2^{nd} job on machine $A = A_1 + A_2$.

Note that process of transportation of job [1] after its completion on machine A and B and processing of 2^{nd} job on machine A and machine B can go simultaneously. Therefore, 2^{nd} job will be ready to be processed on machine B at $[A_1 + A_2 + t_2]$ and on machine C at $[A_1 + A_2 + B_1 + B_2 + t_2 + g_2]$ where as machine C is free at the time $A_1 + t_1 + B_1 + g_1 + C_1$.

Now $A_2+t_2+B_2+g_2 \leq C_1$ using (1) or $A_2+t_2+B_2+g_2 \leq t_1+g_1+C_1$ or $A_1+A_2+t_2+B_1+B_2+g_2 \leq A_1+t_1+B_1+g_1+C_1$. So, $\max\{A_1 + A_2 + t_2 + B_1 + B_2 + g_2,\ \ A_1 + t_1 + B_1 + g_1 + C_1\} = A_1 + t_1 + B_1 + g_1 + C_1$ So, 2^{nd} job will go on machine C at the time $A_1 + t_1 + B_1 + g_1 + C_1$. And will complete on C at time $A_1 + t_1 + B_1 + g_1 + C_1 + C_2$. 3^{rd} job is completed on machine A at the time $= A_1 + A_2 + A_3$, and will be ready for its process on machine B at $A_1 + A_2 + A_3 + t_3$. 3^{rd} job is completed on machine B at the time $= A_1 + A_2 + A_3 + t_3 + B_3$, and will be ready for its process on machine C at $A_1 + A_2 + A_3 + t_3 + B_3 + g_3$.

$$\max\{A_1 + A_2 + A_3 + t_3 + B_3 + g_3,\ \ A_1 + t_1 + B_1 + g_1 + C_1 + C_2\} = A_1 + t_1 + B_1 + g_1 + C_1 + C_2.$$

Therefore, machine C will take 3^{rd} job for processing at $A_1 + t_1 + B_1 + g_1 + C_1 + C_2$. Hence completion time of 3^{rd} job on machine $C = A_1 + t_1 + B_1 + g_1 + C_1 + C_2 + C_3$ and so on.

$$\max\{A_1 + A_2 + \ldots,\ A_i + t_i + B_i + g_i,\ A_1 + t_1 + B_1 + g_1 + C_1 + C_2 + \cdots + C_{i-1}\} = A_1 + t_1 + B_1 + g_1 + C_1 + \cdots + C_{i-1}.$$

Therefore, i^{th} job will complete on machine C at $[A_1 + t_1 + B_1 + g_1 + C_1 + C_2 + \cdots + C_{i-1} + C_i]$ or

$$A_1 + t_1 + B_1 + g_1 + \sum_{j=1}^{[i]} C_j$$

□

Theorem 2.2. *For above stated problem satisfying condition given in (1) without job block, the optimal sequence (minimizing range of lateness) is that in which jobs are both in MST and EDD.*

Proof. By Theorem 2.1, the completion time of i^{th} job on machine C is $[A_1+t_1+B_1+g_1+\sum_{j=1}^{[i]} C_j]$. Let $A_1+t_1+B_1+g_1 = C_1'$. Then completion time of i^{th} job on machine C is

$$[C_1' + \sum_{j=1}^{[i]} C_j] \tag{2}$$

Ikram and Tahir [4] proved in their study that if completion time of i^{th} job on machine C is

$$[A_1 + B_1 + \sum_{j=1}^{[i]} C_j] \tag{3}$$

Then, sequence minimize range of lateness in which jobs are in MST and EDD both terms given in (2) and (3) are same. Therefore, for this problem also, the sequence which minimizes the range of lateness in which jobs are both in MST and EDD that is; the optimal sequence is that in which jobs are in MST and EDD. □

Determination of equivalent job for block of jobs.

Theorem 2.3. *If jobs forming the block have processing time A_{α_k}, $A_{\alpha_{k+1}}$, $A_{\alpha_{k+2}}$ on machine A, B_{α_k}, $B_{\alpha_{k+1}}$, $B_{\alpha_{k+2}}$ on machine B and C_{α_k}, $C_{\alpha_{k+1}}$, $C_{\alpha_{k+2}}$ on machine C. Let β be equivalent job for jobs in block, then processing times of job β on machine A, B and C are given by*

$$A_\beta = A_{\alpha_k} + A_{\alpha_{k+1}}$$

$$B_\beta = B_{\alpha_k} + B_{\alpha_{k+1}} + B_{\alpha_{k+2}} - A_{\alpha_{k+2}}$$

$$C_\beta = C_{\alpha_k} + C_{\alpha_{k+1}} + C_{\alpha_{k+2}} - A_{\alpha_{k+2}}$$

Proof. Maggu and Das [5] proved in their paper that the processing of equivalent job β on machine A, B and C can be calculate by using following formulae. The replacement of jobs in block by β does not increase the completion time.

$$A_\beta = A_{\alpha_k} + A_{\alpha_{k+1}} + A_{\alpha_{k+2}} - \min\{A_{\alpha_{k+2}}, B_{\alpha_{k+1}}, C_{\alpha_k}\}$$

$$B_\beta = B_{\alpha_k} + B_{\alpha_{k+1}} + B_{\alpha_{k+2}} - \min\{A_{\alpha_{k+2}}, B_{\alpha_{k+1}}, C_{\alpha_k}\}$$

$$C_\beta = C_{\alpha_k} + C_{\alpha_{k+1}} + C_{\alpha_{k+2}} - \min\{A_{\alpha_{k+2}}, B_{\alpha_{k+1}}, C_{\alpha_k}\}$$

But $\max\{A_i + t_i + B_i + g_i\} \leq \min\{C_i\}$, therefore
$\min\{A_{\alpha_{k+2}}, B_{\alpha_{k+1}}, C_{\alpha_k}\} = A_{\alpha_{k+2}} \Rightarrow A_\beta = A_{\alpha_k} + A_{\alpha_{k+1}} + A_{\alpha_{k+2}} - A_{\alpha_{k+2}} \Rightarrow$

$$A_\beta = A_{\alpha_k} + A_{\alpha_{k+1}} \tag{4}$$

Now $B_\beta = B_{\alpha_k} + B_{\alpha_{k+1}} + B_{\alpha_{k+2}} - \min\{A_{\alpha_{k+2}}, B_{\alpha_{k+1}}, C_{\alpha_k}\}$. But

$$\max\{A_i + t_i + B_i + g_i\} \leq \min\{C_i\},$$

$$\Rightarrow \max\{A_i + B_i\} \leq \min\{C_i\}$$

$$\Rightarrow \min\{A_{\alpha_{k+2}}, B_{\alpha_{k+1}}, C_{\alpha_k}\} = A_{\alpha_{k+2}}$$

Therefore,

$$B_\beta = B_{\alpha_k} + B_{\alpha_{k+1}} + B_{\alpha_{k+2}} - A_{\alpha_{k+2}} \tag{5}$$

And $C_\beta = C_{\alpha_k} + C_{\alpha_{k+1}} + C_{\alpha_{k+2}} - \min\{A_{\alpha_{k+2}}, B_{\alpha_{k+1}}, C_{\alpha_k}\}$. But $\max\{A_i + t_i + B_i + g_i\} \leq \min\{C_i\} \Rightarrow \max\{A_i + B_i\} \leq \min\{C_i\} \Rightarrow \min\{A_{\alpha_{k+2}}, B_{\alpha_{k+1}}, C_{\alpha_k}\} = A_{\alpha_{k+2}}$. Therefore,

$$C_\beta = C_{\alpha_k} + C_{\alpha_{k+1}} + C_{\alpha_{k+2}} - A_{\alpha_{k+2}} \tag{6}$$

□

On the basis of above 3 Theorems, the following Heuristic Method is developed for the problem defined in this paper with the objective to minimize the range of lateness.

3. Heuristic Method

Step 1: Verify the condition $\max\{A_i + t_i + B_i + g_i\} \leq \min\{C_i\}$ for $i = 1, 2, \ldots, n$.

Step 2: Determine the equivalent job for block of jobs and its processing times of machine A, B, and C by using (4), (5), and (6).

Step 3: Obtain modify problem replacing block of jobs by equivalent job β and its processing times as calculated in step 2. Let t_{α_k}, $t_{\alpha_{k+1}}$, $t_{\alpha_{k+2}}$ and g_{α_k}, $g_{\alpha_{k+1}}$, $g_{\alpha_{k+2}}$ are transportation time and d_{α_k}, $d_{\alpha_{k+1}}$ and $d_{\alpha_{k+2}}$ are due date for jobs in block, then use

$$t_\beta = t_{\alpha_k} + t_{\alpha_{k+1}} + t_{\alpha_{k+2}}$$

$$g_\beta = g_{\alpha_k} + g_{\alpha_{k+1}} + g_{\alpha_{k+2}}$$

$$d_\beta = \min\{d_{\alpha_k} + d_{\alpha_{k+1}} + d_{\alpha_{k+2}}\}$$

Step 4: Arrange the jobs according to MST rule and obtain sequence (s) of jobs.

Step 5: If jobs in sequence obtained in step 4 are also in EDD order then sequence (s) is optimal or next to optimal sequence minimizing the range of lateness.

4. Numerical Example

Consider the following sequencing problem Determine the optimal sequence of jobs which minimize range of lateness with

Table 2. **Required Time, Transportation time and Due dates**

Jobs (i)	Machines A (A_i)	Transportation time (t_i)	Machine B (B_i)	Transportation Time (g_i)	Machine C (C_i)	Due-Date (d_i)
1	2	1	2	3	26	73
2	4	3	3	4	30	80
3	11	1	7	3	25	72
4	8	3	6	7	30	95
5	7	2	3	4	27	75
6	5	5	2	5	31	85
7	10	2	3	6	35	90
8	9	5	4	2	29	80

3 equivalent jobs for block of jobs.

Solution Consider jobs 3, 5, and 8 are in block.

Step 1: $\max\{_Ai + t_i + B_i + g_i\} = \max\{8, 14, 22, 25, 16, 17, 21, 20\} = 25$.

And $\min\{C_i\} = \min\{26, 30, 25, 30, 27, 31, 35, 29\} = 25$. Therefore, $\max\{A_i + t_i + B_i + g_i\} \leq \min\{C_i\}$ for $i = 1, 2, ..., n$.

Step 2: Equivalent job β for jobs 3, 5, and 8. The processing times for β on machine A, machine B and machine C are:

$$A_\beta = A_{\alpha_k} + A_{\alpha_{k+1}} = 11 + 7 = 18$$

$$B_\beta = B_{\alpha_k} + B_{\alpha_{k+1}} + B_{\alpha_{k+2}} - A_{\alpha_{k+2}} = 7 + 3 + 4 - 9 = 5$$

$$C_\beta = C_{\alpha_k} + C_{\alpha_{k+1}} + C_{\alpha_{k+2}} - A_{\alpha_{k+2}} = 25 + 27 + 29 - 9 = 72$$

Step 3: The transportation times and due-date for β are

$$t_\beta = t_{\alpha_k} + t_{\alpha_{k+1}} + t_{\alpha_{k+2}} = 1 + 2 + 5 = 8$$

$$g_\beta = g_{\alpha_k} + g_{\alpha_{k+1}} + g_{\alpha_{k+2}} = 3 + 4 + 2 = 9$$

$$d_\beta = \min\{d_{\alpha_k} + d_{\alpha_{k+1}} + d_{\alpha_{k+2}}\} = \min\{d_3 + d_5 + d_8\} = \min\{72, 75, 80\} = 72$$

So the given table reduces to:

Table 3. **Required Time, Transportation time and Due dates after block job**

Jobs (i)	Machines A (A_i)	Transportation time (t_i)	Machine B (B_i)	Transportation Time (g_i)	Machine C (C_i)	Due-Date (d_i)
1	2	1	2	3	26	73
2	4	3	3	4	30	80
β	18	8	5	9	72	72
4	8	3	6	7	30	95
6	5	5	2	5	31	85
7	10	2	3	6	35	90

Step 4: Slack time of jobs is:

$$d_1 - C_1 = 73 - 26 = 47$$

$$d_2 - C_2 = 80 - 30 = 50$$

$$d_\beta - C_\beta = 72 - 72 = 0$$

$$d_4 - C_4 = 95 - 30 = 65$$

$$d_6 - C_6 = 85 - 31 = 54$$

$$d_7 - C_7 = 90 - 35 = 55$$

According to the Minimum slack time (MST) rule, the job sequence (s) is $\{\beta, 1, 2, 6, 7, 4\}$

Step 5: The Early Due-date (EDD) order for jobs is $\{\beta, 1, 2, 6, 7, 4\}$. Here jobs in sequence obtained in step 4 are also in EDD order then sequence (s) is optimal to sequence which minimizes the range of lateness.

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