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On Multiplicative F-Indices and Multiplicative Connectivity F-Indices of Chemical Networks

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Abstract: The forgotten topological index or F-index is among the most studied topological index, since its applications have been found in Chemistry. In this paper, we compute the multiplicative first and second F-indices, multiplicative first and second hyper F-indices, multiplicative sum connectivity F-index, multiplicative product connectivity F-index, multiplicative atom bond connectivity F-index and multiplicative geometric-arithmetic F-index of armchair polyhex nanotubes, zigzag polyhex nanotubes and carbon nanocone networks.

MSC: 05C05, 05C12, 05C35.

Keywords: Multiplicative F-indices, multiplicative hyper F-indices, multiplicative connectivity indices, network.

1. Introduction

Chemical Graph Theory is a branch of Mathematical Chemistry whose focus of interest is to finding topological indices of a molecular graph which correlate well with chemical properties of the chemical molecules. A molecular graph is a graph such that the vertices correspond to the atoms and the edges to the bonds. A single number that can be used to characterize some property of the graph of molecular is called a topological index. Numerous topological indices have been considered in Theoretical Chemistry, see [1, 2]. Let $G = (V(G), E(G))$ be a finite, simple, connected graph. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. We refer to [3] for undefined term and notation. The first F-index [4] and second F-index [5] of a graph G are defined respectively as

$$F_1(G) = \sum_{uv\in E(G)} \left[d_G(u)^2 + d_G(v)^2\right],$$

$$F_2(G) = \sum_{uv\in E(G)} d_G(u)^2 \, d_G(v)^2.$$

The multiplicative first F-index was introduced by Bhanumati [6] and Ghobadi [7], defined as

$$F_1II(G) = \prod_{uv\in E(G)} \left[d_G(u)^2 + d_G(v)^2\right].$$

We propose the multiplicative second F-index of a graph G and it is defined as

$$F_2II(G) = \prod_{uv\in E(G)} d_G(u)^2 \, d_G(v)^2.$$

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We introduce the multiplicative first and second hyper F-index of a graph G as

$$HF_1II(G) = \prod_{uv \in E(G)} \left[d_G(u)^2 + d_G(v)^2\right]^2,$$
$$HF_2II(G) = \prod_{uv \in E(G)} \left[d_G(u)^2 d_G(v)^2\right]^2.$$

Also we propose the multiplicative sum connectivity F-index and multiplicative product connectivity F-index of a graph G, defined as

$$SFII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)^2 + d_G(v)^2}}.$$
$$PFII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)^2 d_G(v)^2}}.$$

Furthermore, we define the general multiplicative first and second F-indices of a graph G as

$$F_1^a II(G) = \prod_{uv \in E(G)} \left[d_G(u)^2 + d_G(v)^2\right]^a, \quad (1)$$
$$F_2^a II(G) = \prod_{uv \in E(G)} \left[d_G(u)^2 d_G(v)^2\right]^a. \quad (2)$$

We introduce the multiplicative atom bond connectivity F-index of a graph G, defined as

$$ABCFII(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_G(u)^2 + d_G(v)^2 - 2}{d_G(u)^2 d_G(v)^2}}. \quad (3)$$

Finally, we propose the multiplicative geometric-arithmetic F-index of a graph G, defined as

$$GAFII(G) = \prod_{uv \in E(G)} \frac{2\sqrt{d_G(u)^2 d_G(v)^2}}{d_G(u)^2 + d_G(v)^2} \quad (4)$$

In this paper, we consider armchair polyhex nanotubes, zigzag polyhex nanotubes and carbon nanocone networks. Some degree based topological indices of these networks were studied in [8,9]. Multiplicative indices and multiplicative connectivity indices have significant importance to collect information about properties of chemical compounds [2]. These indices related to chemical graph were studied in [10, 11, 12, 13, 14, 15, 16]. In this paper, some topological F-indices of armchair polyhex, zigzag polyhex and carbon nanocone networks are computed.

2. Results for Armchair Polyhex Nanotubes

Carbon nanotubes are well-known allotropes of carbon just like nanocones and fullerenes. So studying carbon nanotube is very important. Carbon polyhex nanotubes are the nanotubes whose cylindrical surface is made up of entirely hexagons. These polyhex nanotubes exist in nature with remarkable stability and poses very interesting electrical, mechanical and thermal properties. We consider the armchair polyhex nanotube which is denoted by $TUAC_6[p, q]$, where p is the number of hexagons in a row and q is the number of hexagons in a column. A 2-dimensional networks of $TUAC_6[p, q]$ is presented in Figure 1.

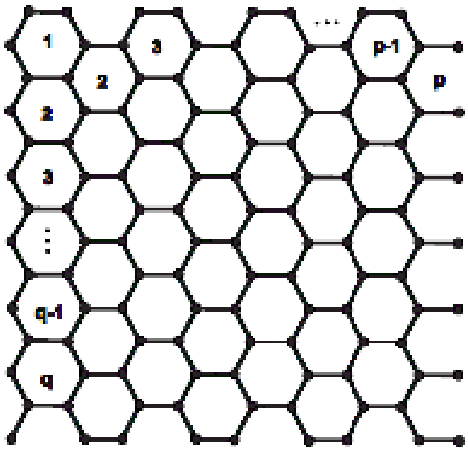

Figure 1: A 2-dimensional networks of $TUAC_6[p,q]$

Let $G = TUAC_6[p,q]$, where $p,q \geq 1$. By calculation, G has $2p(q+1)$ vertices and $3pq+2p$ edges. There are exactly three types of edges based on degrees of end vertices of each edge. We present that the edge partition of G is as follows.

$$E_1 = \{uv \in E(G) | d_G(u) = d_G(v) = 2\}, \qquad |E_1| = p.$$

$$E_2 = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 3\}, \qquad |E_2| = 2p.$$

$$E_3 = \{uv \in E(G) | d_G(u) = d_G(v) = 3\}, \qquad |E_3| = 3pq - p.$$

In the following theorem, we compute the general multiplicative first F-index of $TUAC_6[p,q]$.

Theorem 2.1. *The general multiplicative first F-index of $TUAC_6[p,q]$ is*

$$F_1^a II\left(TUAC_6\left[p,q\right]\right) = 8^{ap} \times 13^{2ap} \times 18^{a(3pq-p)}. \tag{5}$$

Proof. Let $G = TUAC_6[p,q]$. From Equation (1) and by cardinalities of edge partition of G, we have

$$\begin{aligned}
F_1^a II\left(TUAC_6\left[p,q\right]\right) &= \prod_{uv \in E(G)} \left[d_G\left(u\right)^2 + d_G\left(v\right)^2\right]^a \\
&= \left[\left(2^2+2^2\right)^a\right]^p \times \left[\left(2^2+3^2\right)^a\right]^{2p} \times \left[\left(3^2+3^2\right)^a\right]^{3pq-p} \\
&= 8^{ap} \times 13^{2ap} \times 18^{a(3pq-p)}.
\end{aligned}$$

□

We obtain the following results by using Theorem 2.1.

Corollary 2.2. *The multiplicative first F-index of $TUAC_6[p,q]$ is $F_1 II(TUAC_6[p,q]) = 8^p \times 13^{2p} \times 18^{3pq-p}$.*

Proof. Put $a = 1$ in equation (5), we get the desired result. □

Corollary 2.3. *The multiplicative first hyper F-index of $TUAC_6[p,q]$ is $HF_1 II(TUAC_6[p,q]) = 8^{2p} \times 13^{4p} \times 18^{6pq-2p}$.*

Proof. Put $a = 2$ in equation (5), we obtain the desired result. □

Corollary 2.4. *The multiplicative sum connectivity F-index of $TUAC_6[p,q]$ is given by*

$$SFII\left(TUAC_6\left[p,q\right]\right) = \left(\frac{1}{\sqrt{8}}\right)^p \times \left(\frac{1}{\sqrt{13}}\right)^{2p} \times \left(\frac{1}{\sqrt{18}}\right)^{3pq-p}.$$

Proof. Put $a = -\frac{1}{2}$ in equation (5), we get the desired result. $\square$

We now determine the general multiplicative second F-index of $TUAC_6[p, q]$.

Theorem 2.5. *The general multiplicative second F-index of $TUAC_6[p, q]$ is*

$$F_2^a II\left(TUAC_6\left[p, q\right]\right) = 16^{ap} \times 36^{2ap} \times 81^{a(3pq-p)}. \tag{6}$$

Proof. Let $G = TUAC_6[p, q]$. By using Equation (2) and by cardinalities of edge partition of G, we obtain

$$\begin{aligned}
F_2^a II\left(TUAC_6\left[p, q\right]\right) &= \prod_{uv \in E(G)} \left[d_G\left(u\right)^2 d_G\left(v\right)^2\right]^a \\
&= \left[\left(2^2 \times 2^2\right)^a\right]^p \times \left[\left(2^2 \times 3^2\right)^a\right]^{2p} \times \left[\left(3^2 \times 3^2\right)^a\right]^{3pq-p} \\
&= 16^{ap} \times 36^{2ap} \times 81^{a(3pq-p)}.
\end{aligned}$$

$\square$

The following results are obtained by using Theorem 2.5.

Corollary 2.6. *The multiplicative second F-index of $TUAC_6[p, q]$ is given by $F_2II(TUAC_6[p, q]) = 16^p \times 36^{2p} \times 81^{3pq-p}$.*

Proof. Put $a = 1$ in Equation (6), we get the desired result. $\square$

Corollary 2.7. *The multiplicative second hyper F-index of $TUAC_6[p, q]$ is given by $HF_2II(TUAC_6[p, q]) = 16^{2p} \times 36^{4p} \times 81^{6pq-2p}$.*

Proof. Put $a = 2$, in Equation (6), we get the desired result. $\square$

Corollary 2.8. *The multiplicative product connectivity F-index of $TUAC_6[p, q]$ is*

$$PF\left(TUAC_6\left[p, q\right]\right) = \left(\frac{1}{4}\right)^p \left(\frac{1}{6}\right)^{2p} \left(\frac{1}{9}\right)^{3pq-p}.$$

Proof. Put $a = -\frac{1}{2}$ in Equation (6), we obtain the desired result. $\square$

In the following theorems, we compute the multiplicative atom bond connectivity F-index and multiplicative geometric-arithmetic F-index of $TUAC_6[p, q]$.

Theorem 2.9. *The multiplicative atom bond connectivity F-index of $TUAC_6[p, q]$ is*

$$ABCFII\left(TUAC6\left[p, q\right]\right) = \left(\sqrt{\frac{3}{8}}\right)^p \times \left(\frac{11}{36}\right)^p \times \left(\frac{4}{9}\right)^{3pq-p}.$$

Proof. Let $G = TUAC_6[p, q]$. From Equation (3) and by cardinalities of edge partition of G, we deduce

$$\begin{aligned}
ABCFII\left(TUAC_6\left[p, q\right]\right) &= \prod_{uv \in E(G)} \sqrt{\frac{d_G\left(u\right)^2 + d_G\left(v\right)^2 - 2}{d_G\left(u\right)^2 d_G\left(v\right)^2}} \\
&= \left(\sqrt{\frac{2^2 + 2^2 - 2}{2^2 \times 2^2}}\right)^p \times \left(\sqrt{\frac{2^2 + 3^2 - 2}{2^2 \times 3^2}}\right)^{2p} \times \left(\sqrt{\frac{3^2 + 3^2 - 2}{3^2 \times 3^2}}\right)^{3pq-p} \\
&= \left(\sqrt{\frac{3}{8}}\right)^p \times \left(\frac{11}{36}\right)^p \times \left(\frac{4}{9}\right)^{3pq-p}.
\end{aligned}$$

$\square$

Theorem 2.10. *The multiplicative geometric-arithmetic F-index of $TUAC_6[p,q]$ is given by*

$$GAFII\left(TUAC_6\left[p,q\right]\right) = \left(\frac{12}{13}\right)^{2P}.$$

Proof. Let $G = TUAC_6\left[p,q\right]$. From Equation (4) and by cordialities of edge partition of G, we derive

$$\begin{aligned} GAFII\left(TUAC_6\left[p,q\right]\right) &= \prod_{uv\in E(G)} \frac{2\sqrt{d_G\left(u\right)^2 d_G\left(v\right)^2}}{d_G\left(u\right)^2 + d_G\left(v\right)^2} \\ &= \left(\frac{2\sqrt{2^2\times 2^2}}{2^2+2^2}\right)^{p} \times \left(\frac{2\sqrt{2^2\times 3^2}}{2^2+3^2}\right)^{2p} \times \left(\frac{2\sqrt{3^2\times 3^2}}{3^2+3^2}\right)^{3pq-p} \\ &= \left(\frac{12}{13}\right)^{2p}. \end{aligned}$$

□

3. Results for Zigzag Polyhex Nanotudes

We consider the zigzag polyhex nanotube which is denoted by $TUAC_6\left[p,q\right]$, where p is the number of hexagons in a row and q is the number of hexagons in a column. A 2-dimensional networks of $TUAC_6\left[p,q\right]$ is shown in Figure 2.

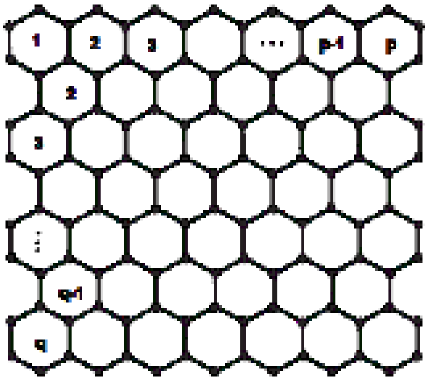

Figure 2: A 2-dimensional networks of $TUZC_6[p,q]$

Let $G = TUZC_6\left[p,q\right]$, where $p,q \geq 1$. By calculation, G has $2p(q+1)$ vertices and $3pq+2p$ edges. In G, there are two types of edges based on degrees of end vertices of each edge. The edge partition of G is as follows:

$$E_1 = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 3\}, \qquad |E_1| = 4p.$$

$$E_2 = \{uv \in E(G) | d_G(u) = d_G(v) = 3\}, \qquad |E_2| = 3pq - 2p.$$

In the following theorem, we determine the general multiplicative first F-index of $TUZC_6\left[p,q\right]$.

Theorem 3.1. *The general multiplication first F-index of $TUZC_6\left[p,q\right]$ is*

$$F_1^a II\left(TUZC_6\left[p,q\right]\right) = 13^{4ap} \times 18^{a(3pq-2p)}. \tag{7}$$

Proof. Let $G = TUZC_6\left[p,q\right]$. By using Equation (1) and by cardinalities of edge partition of G, we deduce

$$\begin{aligned} F_1^a II\left(TUZC_6\left[p,q\right]\right) &= \prod_{uv\in E(G)} \left[d_G\left(u\right)^2 + d_G\left(v\right)^2\right]^a \\ &= \left[\left(2^2+3^2\right)^a\right]^{4p} \times \left[\left(3^2+3^2\right)^a\right]^{3pq-2p} \\ &= 13^{4ap} \times 18^{a(3pq-2p)}. \end{aligned}$$

□

We establish the following results by using Theorem 3.1.

Corollary 3.2. *The multiplicative first F-index of* $TUZC_6\,[p,q]$ *is* $F_1II\,(TUZC_6\,[p,q]) = 13^{4p} \times 18^{3pq-2p}$.

Proof. Put $a = 1$ is Equation (7), we get the desired result. □

Corollary 3.3. *The multiplicative first hyper F-index of* $TUZC_6\,[p,q]$ *in* $HF_1II\,(TUZC_6\,[p,q]) = 13^{8p} \times 18^{6pq-4p}$.

Proof. Put $a = 2$ is Equation (7), we get the desired result. □

Corollary 3.4. *The multiplicative sum connectivity F-index of* $TUZC_6\,[p,q]$ *in* $SFII\,(TUAC_3\,[p,q]) = \left(\frac{1}{\sqrt{13}}\right)^{4p} \times \left(\frac{1}{\sqrt{18}}\right)^{3pq-2p}$.

Proof. Put $a = -\frac{1}{2}$ in Equation (7), we get the desired result. □

In the following theorem, we compute the general multiplicative second F-index of $TUZC_6\,[p,q]$.

Theorem 3.5. *The general multiplicative second F-index of* $TUZC_6\,[p,q]$ *is given by*

$$F_2^aII\,(TUZC_6\,[p,q]) = 36^{4ap} \times 81^{a(3pq-2p)}. \tag{8}$$

Proof. Let $G = TUZC_6\,[p,q]$. From Equation (2) and by cardinalities of the edge partition of G, we derive

$$\begin{aligned} F_2^aII\,(TUZC_6\,[p,q]) &= \prod_{uv\in E(G)} \left[d_G(u)^2\, d_G(v)^2\right]^a \\ &= \left[\left(2^2 \times 3^2\right)^a\right]^{4p} \times \left[\left(3^2 \times 3^2\right)^a\right]^{3pq-2p} \\ &= 36^{4ap} \times 81^{a(3pq-2p)}. \end{aligned}$$

□

We obtain the following results from Theorem 3.5.

Corollary 3.6. *The multiplicative second F-index of* $TUZC_6\,[p,q]$ *is* $F_2II\,(TUZC_6\,[p,q]) = 36^{4p} \times 81^{3pq-2p}$.

Proof. Put $a = 1$ in Equation (8), we obtain the desired result. □

Corollary 3.7. *The multiplicative second F-index of* $TUZC_6\,[p,q]$ *is* $HF_2II\,(TUZC_6\,[p,q]) = 36^{8p} \times 81^{6pq-4p}$.

Proof. Put $a = 2$ in Equation (8), we get the desired result. □

Corollary 3.8. *The multiplicative product connectivity F-index of* $TUZC_6\,[p,q]$ *is given by* $PFII\,(TUZC_6\,[p,q]) = \left(\frac{1}{6}\right)^{4p} \times \left(\frac{1}{9}\right)^{3pq-2p}$.

Proof. Put $a = -\frac{1}{2}$ in Equation (8), we get the desired result. □

We determine the multiplicative atom bond connectivity F-index and multiplicative geometric-arithmetic F-index of $TUZC_6\,[p,q]$.

Theorem 3.9. *The multiplicative atom bond connectivity F-index of* $TUZC_6\,[p,q]$ *is*

$$ABCFII\,(TUZC6\,[p,q]) = \left(\frac{11}{36}\right)^{2p} \times \left(\frac{4}{9}\right)^{3pq-2p}.$$

Proof. Let $G = TUZC_6[p,q]$. By using Equation (3) and by cardinalities of edge partition of G, we have

$$\begin{aligned} ABCFII\left(TUZC_6\left[p,q\right]\right) &= \prod_{uv\in E(G)} \sqrt{\frac{d_G\left(u\right)^2+d_G\left(v\right)^2-2}{d_G\left(u\right)^2 d_G\left(v\right)^2}} \\ &= \left(\sqrt{\frac{2^2+3^2-2}{2^2\times 3^2}}\right)^{4p} \times \left(\sqrt{\frac{3^2+3^2-2}{3^2\times 3^2}}\right)^{3pq-2p} \\ &= \left(\frac{11}{36}\right)^{2p} \times \left(\frac{4}{9}\right)^{3pq-2p}. \end{aligned}$$

□

Theorem 3.10. *The multiplicative geometric-arithmetic F-index of* $TUZC_6\left[p,q\right]$ *is given by*

$$GAFII\left(TUZC_6\left[p,q\right]\right) = \left(\frac{12}{13}\right)^{4P}.$$

Proof. Let $G = TUZC_6\left[p,q\right]$. From Equation (4) and by cardinalities of the edge partition of G we obtain

$$\begin{aligned} GAFII\left(TUZC_6\left[p,q\right]\right) &= \prod_{uv\in E(G)} \frac{2\sqrt{d_G\left(u\right)^2 d_G\left(v\right)^2}}{d_G\left(u\right)^2+d_G\left(v\right)^2} \\ &= \left(\frac{2\sqrt{2^2\times 3^2}}{2^2+3^2}\right)^{4p} \times \left(\frac{2\sqrt{3^2\times 3^2}}{3^2+3^2}\right)^{3pq-2p} \\ &= \left(\frac{12}{13}\right)^{4p}. \end{aligned}$$

□

4. Results for Carbon Nanocone Networks

We consider an n-dimensional one pentagonal nanocone, which is symbolized by $CNC_5[n]$, where n is the number of hexagons layers encompassing the canonical surface of the nanocone and 5 denotes that there is a pentagon on the tip called its core. A 6-dimensional one pentagonal nanocone network is presented in Figure 3.

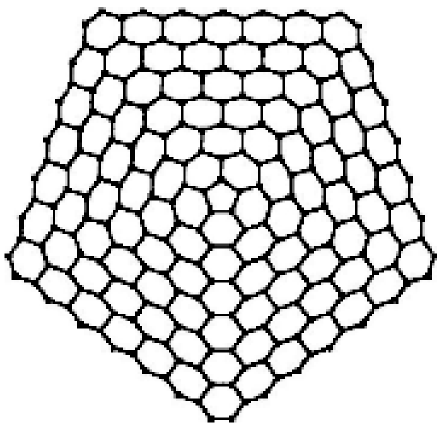

Figure 3: A 6-dimensional one pentagonal nanocone network

Let G be the n-dimensional one-pentagonal nanocone network $CNC_5[n]$, $n \geq 2$. By calculation, G has $5(n+1)^2$ vertices and $\frac{15}{2}n^2 + \frac{25}{2}n + 5$ edges. In G, there are exactly three types of edges based on the degrees of end vertices of each edge. This edge partition is given below.

$$E_1 = \{uv \in E(G) | d_G(u) = d_G(v) = 2\}, \qquad |E_1| = 5.$$

$$E_2 = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 3\}, \qquad |E_2| = 10n.$$

$$E_3 = \{uv \in E(G) | d_G(u) = d_G(v) = 3\}, \qquad |E_3| = \frac{15}{2}n^2 + \frac{5}{2}n.$$

In the following theorem, we compute the general multiplicative first F-index of $CNC_5[n]$.

Theorem 4.1. *The general multiplicative first F-index of $CNC_5[n]$ is given by*

$$F_1^a II\left(CNC_5\left[n\right]\right) = 8^{5a} \times 13^{10an} \times 18^{a\left(\frac{15}{2}n^2+\frac{5}{2}n\right)}. \tag{9}$$

Proof. Let $G = CNC_5[n]$. By using Equation (1) and by cardinalities of edge partition of G, we obtain

$$\begin{aligned} F_1^a II\left(CNC_5\left[n\right]\right) &= \prod_{uv\in E(G)} \left[d_G\left(u\right)^2 + d_G\left(v\right)^2\right]^a \\ &= \left[\left(2^2+2^2\right)^a\right]^5 \times \left[\left(2^2+3^2\right)^a\right]^{10n} \times \left[\left(3^2+3^2\right)^a\right]^{\frac{15}{2}n^2+\frac{5}{2}n} \\ &= 8^{5a} \times 13^{10an} \times 18^{a\left(\frac{15}{2}n^2+\frac{5}{2}n\right)}. \end{aligned}$$

□

The following results are obtained by using Theorem 4.1.

Corollary 4.2. *The multiplicative first F-index of $CNC_5[n]$ is* $F_1 II\left(CNC_5\left[n\right]\right) = 8^5 \times 13^{10n} \times 18^{\frac{15}{2}n^2+\frac{5}{2}n}$.

Proof. Put $a = 1$ in Equation (9), we get the desired result. □

Corollary 4.3. *The multiplicative first hyper F-index of $CNC_5[n]$ is* $HF_1 II\left(CNC_5\left[n\right]\right) = 8^{10} \times 13^{20n} \times 18^{15n^2+5n}$.

Proof. Put $a = 2$ in Equation (9), we obtain the desired result. □

Corollary 4.4. *The multiplicative sum connectivity F-index of $CNC_5[n]$*

$$SFII\left(CNC_5\left[n\right]\right) = \left(\frac{1}{\sqrt{8}}\right)^5 \times \left(\frac{1}{\sqrt{13}}\right)^{10n} \times \left(\frac{1}{\sqrt{18}}\right)^{\frac{15}{2}n^2+\frac{5}{2}n}.$$

Proof. Put $a = -\frac{1}{2}$ in Equation (9), we get the desired result. □

We now determine the general multiplicative second F-index of $CNC_5[n]$.

Theorem 4.5. *The general multiplicative second F-index of $CNC_5[n]$ is given by*

$$F_2^a II\left(CNC_5\left[n\right]\right) = 16^{5a} \times 36^{10an} \times 81^{\left(\frac{15}{2}n^2+\frac{5}{2}n\right)}. \tag{10}$$

Proof. Let $G = CNC_5[n]$. From Equation (2) and by cardinalities of edge position of G, we deduce

$$\begin{aligned} F_2^a II\left(CNC_5\left[n\right]\right) &= \prod_{uv\in E(G)} \left[d_G\left(u\right)^2 d_G\left(v\right)^2\right]^a \\ &= \left[\left(2^2\times 2^2\right)^a\right]^5 \times \left[\left(2^2\times 3^2\right)^a\right]^{10n} \times \left[\left(3^2\times 3^2\right)^a\right]^{\frac{15}{2}n^2+\frac{5}{2}n} \\ &= 16^{5a} \times 36^{10an} \times 81^{a\left(\frac{15}{2}n^2-\frac{5}{2}n\right)}. \end{aligned}$$

□

We get the following results by using Theorem 4.5.

Corollary 4.6. *The multiplicative second F-index of $CNC_5[n]$ is* $F_2 II\left(CNC_5\left[n\right]\right) = 16^5 \times 36^{10n} \times 81^{\frac{15}{2}n^2+\frac{5}{2}n}$.

Proof. Put $a = 1$ in Equation (10), we obtain the desired result. □

Corollary 4.7. *The multiplicative second hyper F-index of $CNC_5[n]$ is* $HF_2II\left(CNC_5\left[n\right]\right) = 16^{10} \times 36^{20n} \times 81^{15n^2+5n}$.

Proof. Put $a = 2$ in Equation (10), we get the desired result. □

Corollary 4.8. *The multiplicative product connectivity F-index of $CNC_5[n]$ is given by*

$$PFII\left(CNC_5\left[n\right]\right) = \left(\frac{1}{4}\right)^5 \times \left(\frac{1}{6}\right)^{10n} \times \left(\frac{1}{9}\right)^{\frac{15}{2}n^2+\frac{5}{2}n}.$$

Proof. Put $a = -\frac{1}{2}$ in Equation (10), we obtain the desired result. □

In the following theorems, we compute the multiplicative atom bond connectivity F-index and multiplicative geometric-arithmetic F-index of $CNC_5[n]$.

Theorem 4.9. *The multiplicative atom bond connectivity F-index of $CNC_5[n]$ is given by*

$$ABCFII\left(CNC_5\left[n\right]\right) = \left(\sqrt{\frac{3}{8}}\right)^5 \times \left(\frac{11}{36}\right)^{10n} \times \left(\frac{2}{3}\right)^{30n^2+10n}.$$

Proof. Let $G = CNC_5[n]$. From Equation (3) and by cardinalities of edge position of G, we derive

$$\begin{aligned}
ABCFII\left(CNC_5\left[n\right]\right) &= \prod_{uv\in E(G)} \sqrt{\frac{d_G\left(u\right)^2 + d_G\left(v\right)^2 - 2}{d_G\left(u\right)^2 d_G\left(v\right)^2}} \\
&= \left(\sqrt{\frac{2^2+2^2-2}{2^2\times 2^2}}\right)^5 \times \left(\sqrt{\frac{2^2+3^2-2}{2^2\times 3^2}}\right)^{10n} \times \left(\sqrt{\frac{3^2+3^2-2}{3^2\times 3^2}}\right)^{\frac{15}{2}n^2+\frac{5}{2}n} \\
&= \left(\sqrt{\frac{3}{8}}\right)^5 \times \left(\frac{11}{36}\right)^{5n} \times \left(\frac{2}{3}\right)^{30n^2+10n}.
\end{aligned}$$

□

Theorem 4.10. *The multiplicative geometric-arithmetic F-index of $CNC_5[n]$ is*

$$GAFII\left(CNC_5\left[n\right]\right) = \left(\frac{12}{13}\right)^{10n}.$$

Proof. Let $G = CNC_5[n]$. By using Equation (4) and by cardinalities of edge partition of G , we deduce

$$\begin{aligned}
GAFII\left(CNC_5\left[n\right]\right) &= \prod_{uv\in E(G)} \frac{2\sqrt{d_G\left(u\right)^2 d_G\left(v\right)^2}}{d_G\left(u\right)^2 + d_G\left(v\right)^2} \\
&= \left(\frac{2\sqrt{2^2\times 2^2}}{2^2+2^2}\right)^5 \times \left(\frac{2\sqrt{2^2\times 3^2}}{2^2+3^2}\right)^{10n} \times \left(\frac{2\sqrt{3^2\times 3^2}}{3^2+3^2}\right)^{\frac{15}{2}n^2+\frac{5}{2}n} \\
&= \left(\frac{12}{13}\right)^{10n}.
\end{aligned}$$

□

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