



Hyper-Revan Indices and their Polynomials of Silicate Networks

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Abstract: We propose the first and second hyper-Revan indices of a molecular graph. Considering these hyper-Revan indices, we define the first and second hyper-Revan polynomials of a graph. In this paper, we compute the first and second hyper-Revan indices and their polynomials of certain family of networks such as silicate networks.

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1. Introduction

A molecular graph is a graph whose vertices correspond to the atoms and the edges to the bonds, Chemical graph theory has an important effect on the development of Chemical Sciences, see [1]. Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v . Let $\Delta(G)$ ($\delta(G)$) denote the maximum (minimum) degree among the vertices of G . The revan vertex degree of a vertex v in G is defined as $r_G(v) = \Delta(G) + \delta(G) - d_G(v)$. The revan edge connecting the revan vertices u and v will be denoted by uv . We refer [2], for other undefined notations and terminologies. The first and second Revan indices of a graph G are defined as

$$R_1(G) = \sum_{uv \in E(G)} [r_G(u) + r_G(v)], \quad R_2(G) = \sum_{uv \in E(G)} r_G(u) r_G(v).$$

The Revan indices were introduced by Kulli in [3] and were studied, for example, in [4, 5]. Considering the Revan indices, the first and second Ravan polynomials [6] of a graph were defined as follows:

The first and second Revan polynomials of a graph G are defined as

$$R_1(G, x) = \sum_{uv \in E(G)} x^{r_G(u) + r_G(v)}, \quad R_2(G, x) = \sum_{uv \in E(G)} x^{r_G(u) r_G(v)}.$$

We introduce the first and second hyper-Revan indices as

$$HR_1(G) = \sum_{uv \in E(G)} [r_G(u) + r_G(v)]^2, \quad HR_2(G) = \sum_{uv \in E(G)} [r_G(u) r_G(v)]^2.$$

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Considering hyper-Revan indices, we propose the first and second hyper-Revan polynomials of a graph G as

$$HR_1(G, x) = \sum_{uv \in E(G)} x^{[r_G(u) + r_G(v)]^2}, \quad HR_2(G, x) = \sum_{uv \in E(G)} x^{[r_G(u)r_G(v)]^2}.$$

The third Revan index [3] of a graph G is defined as

$$R_3(G) = \sum_{uv \in E(G)} |r_G(u) - r_G(v)|.$$

Considering the third Revan index, Kulli defined the third Revan polynomial [5] as

$$R_3(G, x) = \sum_{uv \in E(G)} x^{|r_G(u) - r_G(v)|}.$$

Recently, several topological indices were studied, for example, in [7–17]. In this paper, the first and second hyper-Revan indices and their polynomials of silicate networks are determined. For silicate networks see [5].

2. Results

A silicate network is symbolized by SL_n where n is the number of hexagons between the center and boundary of SL_n . These networks are obtained by fusing metal oxides or metal carbonates with sand. A silicate network of dimension 2 is depicted in Figure 1.

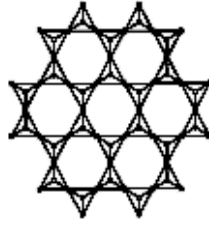


Figure 1: A 2-dimensional silicate network

Let G be the graph of silicate network SL_n . By calculation, G has $15n^2 + 3n$ vertices and $36n^2$ edges. From Figure 1, it is easy to see that the vertices of SL_n are either of degree 3 or 6. Thus $\Delta(G) = 6$, $\delta(G) = 3$ and therefore $r_G(u) = 9 - d_G(u)$. In SL_n , by algebraic method, there are three types of edges based on the degree of the vertices of each edge as follows:

$$E_{33} = \{uv \in E(G) | d_G(u) = d_G(v) = 3\}, \quad |E_{33}| = 6n.$$

$$E_{36} = \{uv \in E(G) | d_G(u) = 3, d_G(v) = 6\}, \quad |E_{36}| = 18n^2 + 6n.$$

$$E_{66} = \{uv \in E(G) | d_G(u) = d_G(v) = 6\}, \quad |E_{66}| = 18n^2 - 12n.$$

Thus there are three types of Revan edges based on the degree of end Revan vertices of each Revan edge as given in Table 1.

$r_G(u), r_G(v) / uv \in E(G)$	(6,6)	(6, 3)	(3,3)
Number of edges	6n	$18n^2 + 6n$	$18n^2 - 12n$

Table 1: Revan edge partition of G

In the following theorem, we compute the value of $R_1(SL_n, x)$, $R_2(SL_n, x)$ for silicate networks.

Theorem 2.1. *The first and second Revan polynomials of a silicate network are given by*

$$(1). R_1(SL_n, x) = 6nx^{12} + (18n^2 + 6n)x^9 + (18n^2 - 12n)x^6.$$

$$(2). R_2(SL_n, x) = 6nx^{36} + (18n^2 + 6n)x^{18} + (18n^2 - 12n)x^9.$$

Proof. Let G be the graph of silicate network SL_n .

(1). By using the partition given in Table 1, we can apply the formula of the first Ravan Polynomial of silicate network SL_n .

Since $R_1(G, x) = \sum_{uv \in E(G)} x^{r_G(u) + r_G(v)}$, this implies that

$$\begin{aligned} R_1(SL_n, x) &= 6nx^{6+6} + (18n^2 + 6n)x^{6+3} + (18n^2 - 12n)x^{3+3}. \\ &= 6nx^{12} + (18n^2 + 6n)x^9 + (18n^2 - 12n)x^6. \end{aligned}$$

(2). By using the partition given Table 1, we can apply the formula of the second Revan Polynomial of silicate network SL_n .

Since $R_2(G, x) = \sum_{uv \in E(G)} x^{r_G(u)r_G(v)}$, this implies that

$$\begin{aligned} R_2(SL_n, x) &= 6nx^{6 \times 6} + (18n^2 + 6n)x^{6 \times 3} + (18n^2 - 12n)x^{3 \times 3}. \\ &= 6nx^{36} + (18n^2 + 6n)x^{18} + (18n^2 - 12n)x^9. \end{aligned}$$

□

In the following theorem, we compute the third Revan index and its polynomial of silicate networks.

Theorem 2.2. *The third Revan index and its polynomial of a silicate network are given by*

$$(1). R_3(SL_n) = 54n^2 + 18n.$$

$$(2). R_3(SL_n, x) = (18n^2 + 6n)x^3 + (18n^2 - 6n).$$

Proof. Let G be the graph of silicate network SL_n .

(1). By using the partition given in Table 1, we can apply the formula of the third Revan index of silicate network SL_n .

Since $R_3(G) = \sum_{uv \in E(G)} |r_G(u) - r_G(v)|$, this implies that

$$\begin{aligned} R_3(SL_n) &= 6n \times 0 + (18n^2 + 6n)3 + (18n^2 - 12n) \times 0. \\ &= 54n^2 + 18n. \end{aligned}$$

(2). By using the partition given in Table 1, we can apply the formula of the third Revan Polynomial of silicate network

SL_n . Since $R_3(G, x) = \sum_{uv \in E(G)} x^{|r_G(u) - r_G(v)|}$, this implies that

$$\begin{aligned} R_3(SL_n, x) &= 6nx^0 + (18n^2 + 6n)x^3 + (18n^2 - 12n)x^0. \\ &= (18n^2 + 6n)x^3 + (18n^2 - 6n). \end{aligned}$$

□

In the following theorem, we compute the value of $HR_1(SL_n)$ and $HR_2(SL_n)$ for silicate networks.

Theorem 2.3. *The first and second hyper-Revan indices of a silicate network are given by*

$$(1). HR_1(SL_n) = 2106n^2 + 918n.$$

$$(2). HR_2(SL_n) = 7290n^2 + 8748n.$$

Proof. Let G be the graph of silicate network SL_n .

(1). By using the partition given in Table 1, we can apply the formula of the first hyper-Revan index of G . Since $HR_1(G) =$

$$\sum_{uv \in E(G)} [r_G(u) + r_G(v)]^2, \text{ this implies that}$$

$$\begin{aligned} HR_1(SL_n) &= 6n(6+6)^2 + (18n^2 + 6n)(6+3)^2 + (18n^2 - 12n)(3+3)^2 \\ &= 2106n^2 + 918n. \end{aligned}$$

(2). By using the partition given in Table 1, we can apply the formula of the second hyper-Revan index of G . Since

$$HR_2(G) = \sum_{uv \in E(G)} [r_G(u) r_G(v)]^2, \text{ this implies that}$$

$$\begin{aligned} HR_2(SL_n) &= 6n(6 \times 6)^2 + (18n^2 + 6n)(6 \times 3)^2 + (18n^2 - 12n)(3 \times 3)^2 \\ &= 7290n^2 + 8748n. \end{aligned}$$

□

In the following theorem, we compute the first and second hyper-Revan polynomials of silicate networks.

Theorem 2.4. *The first and second hyper-Revan Polynomials of a silicate network are given by*

$$(1). HR_1(SL_n, x) = 6nx^{144} + (18n^2 + 6n)x^{81} + (18n^2 - 12n)x^{36}.$$

$$(2). HR_2(SL_n, x) = 6nx^{1296} + (18n^2 + 6n)x^{328} + (18n^2 - 12n)x^{81}.$$

Proof. Let G be the graph of silicate network SL_n .

(1). By using the partition given Table 1, we can apply the formula of the first hyper-Revan polynomial of a silicate network

$$SL_n. \text{ Since } HR_1(G, x) = \sum_{uv \in E(G)} x^{[r_G(u)+r_G(v)]^2}, \text{ this implies that}$$

$$\begin{aligned} HR_1(SL_n, x) &= 6nx^{(6+6)^2} + (18n^2 + 6n)x^{(6+3)^2} + (18n^2 - 12n)x^{(3+3)^2} \\ &= 6nx^{144} + (18n^2 + 6n)x^{81} + (18n^2 - 12n)x^{36}. \end{aligned}$$

(2). By using the partition given in Table 1, we can apply the formula of the second hyper-Revan polynomial of a silicate

$$\text{network } SL_n. \text{ Since } HR_2(G, x) = \sum_{uv \in E(G)} x^{[r_G(u)r_G(v)]^2}, \text{ this implies that}$$

$$\begin{aligned} HR_2(SL_n, x) &= 6nx^{(6 \times 6)^2} + (18n^2 + 6n)x^{(6 \times 3)^2} + (18n^2 - 12n)x^{(3 \times 3)^2} \\ &= 6nx^{1296} + (18n^2 + 6n)x^{324} + (18n^2 - 12n)x^{81}. \end{aligned}$$

□

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