International Journal of Current Research in Science and Technology
Volume 3, Issue 10 (2017), 29–33.
ISSN: 2394-5745
Available Online: http://ijcrst.in/

International Journal of Current Research in Science and Technology

Geometric-Arithmetic Reverse and Sum Connectivity Reverse Indices of Silicate and Hexagonal Networks

Research Article

V.R.Kulli[1,*]

1 Department of Mathematics, Gulbarga University, Gulbarga, Karnataka, India.

Abstract: We introduce a new index known as geometric-arithmetic reverse index of a molecular graph. In this paper, we compute geometric-arithmetic reverse index and sum connectivity reverse index of different chemically interesting networks like silicate networks and hexagonal networks.

MSC: 05C05, 05C12, 05C35.

Keywords: Geometric-arithmetic reverse index, sum connectivity reverse index, silicate network, hexagonal network.

1. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. Let $d_G(v)$ denote the degree of a vertex v in G. Let $\Delta(G)$ denote the maximum degree among the vertices of G. The reverse vertex degree of a vertex u in G is defined as $c_u = \Delta(G) - d_G(u) + 1$. The reverse edge connecting the reverse vertices u and v will be denoted by uv. For all further notation and terminology we refer to reader to [1]. Chemical graph theory is a branch of Mathematical Chemistry which has an important effect on the development of the chemical sciences. A topological index is a numerical parameter mathematically derived from the graph structure. Numerous such topological indices have been considered in Theoretical Chemistry. Recently we introduced the atom bond connectivity reverse index [2] of a graph G as

$$ABCC(G) = \sum_{uv \in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}}.$$

The sum connectivity reverse index was introduced by Kulli in [3]. The sum connectivity reverse index of a graph G is defined as

$$SC(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{c_u + c_v}}.$$

Recently some reverse indices were studied, for example, in [4–6]. We now introduce the geometric-arithmetic reverse index of a graph G as

$$GAC(G) = \sum_{uv \in E(G)} \frac{2\sqrt{c_u c_v}}{c_u + c_v}. \tag{1}$$

Recently several topological indices were studied, for example, in [7–17]. In this paper, the geometric-arithmetic reverse index and sum connectivity reverse index of silicate networks and hexagonal networks are computed. For silicate networks and hexagonal networks see [18].

* E-mail: vrkulli@gmail.com

2. Results for Silicate Networks

Silicates are obtained by fusing metal oxide or metal carbonates with sand. A silicate network is symbolized by SL_n where n is the number of hexagons between the center and boundary of SL_n. A 2-dimensional silicate network is depicted in Figure 1.

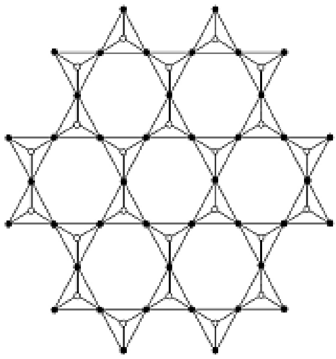

Figure 1: A 2-dimensional silicate network

Let G be the graph of silicate network SL_n. From Figure 1, it is easy to see that the vertices of SL_n are either of degree 3 or 6. Then $\Delta(G) = 6$. By algebraic method, we obtain that $|V(SL_n)| = 15n^2 + 3n$ and $|E(SL_n)| = 36n^2$. In SL_n, by algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$$E_{33} = \{uv \in E(G) | d_G(u) = d_G(v) = 3\}, \quad |E_{33}| = 6n.$$

$$E_{36} = \{uv \in E(G) | d_G(u) = 3, d_G(v) = 6\}, \quad |E_{36}| = 18n^2 + 6n.$$

$$E_{66} = \{uv \in E(G) | d_G(u) = d_G(v) = 6\}, \quad |E_{66}| = 18n^2 - 12n.$$

We have $c_u = \Delta(G) - d_G(u) + 1 = 7 - d_G(u)$. Thus there are three types of reverse edges of follows:

$$CE_{44} = \{uv \in E(G) | c_u = c_v = 4\}, \quad |CE_{44}| = 6n.$$

$$CE_{41} = \{uv \in E(G) | c_u = 4, c_v = 1\}, \quad |CE_{41}| = 18n^2 + 6n.$$

$$CE_{11} = \{uv \in E(G) | c_u = c_v = 1\}, \quad |CE_{11}| = 18n^2 - 12n.$$

In the following theorem, we compute the geometric-arithmetic reverse index of silicate networks.

Theorem 2.1. *The geometric-arithmetic reverse index of silicate networks is given by*

$$GAC\,(SL_n) = \frac{162}{5}n^2 - \frac{6}{5}n.$$

Proof. By definition, we have

$$GAC\,(G) = \sum_{uv \in E(G)} \frac{2\sqrt{c_u c_v}}{c_u + c_v}$$

Thus

$$\begin{aligned} GAC\,(SL_n) &= \sum_{CE_{44}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{41}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{11}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} \\ &= \left(\frac{2\sqrt{4 \times 4}}{4+4}\right) 6n + \left(\frac{2\sqrt{4 \times 1}}{4+1}\right)(18n^2 + 6n) + \left(\frac{2\sqrt{1 \times 1}}{1+1}\right)(18n^2 - 12n) \\ &= \frac{162}{5}n^2 - \frac{6}{5}n. \end{aligned}$$

□

In the following theorem, we compute the sum connectivity reverse index of silicate networks.

Theorem 2.2. *The sum connectivity reverse index of silicate networks is given by*

$$SC\left(SL_n\right) = \left(\frac{18}{\sqrt{5}} + \frac{18}{\sqrt{2}}\right) n^2 + \left(\frac{6}{\sqrt{5}} - \frac{9}{\sqrt{2}}\right) n.$$

Proof. By definition, we have

$$SC\left(G\right) = \sum_{uv \in E(G)} \frac{1}{\sqrt{c_u + c_v}}.$$

Thus

$$\begin{aligned} SC\left(SL_n\right) &= \sum_{CE_{44}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{41}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{11}} \frac{1}{\sqrt{c_u + c_v}} \\ &= \left(\frac{1}{\sqrt{4+4}}\right) 6n + \left(\frac{1}{\sqrt{4+1}}\right)\left(18n^2 + 6n\right) + \left(\frac{1}{\sqrt{1+1}}\right)\left(18n^2 - 12n\right) \\ &= \left(\frac{18}{\sqrt{5}} + \frac{18}{\sqrt{2}}\right) n^2 + \left(\frac{6}{\sqrt{5}} - \frac{9}{\sqrt{2}}\right) n. \end{aligned}$$

□

3. Results for Hexagonal Networks

A triangular tiling is used in the construction of hexagonal networks. A hexagonal network is symbolized by HX_n where n is the number of vertices in each side of hexagon. A hexagonal network of dimension six is depicted in Figure 2.

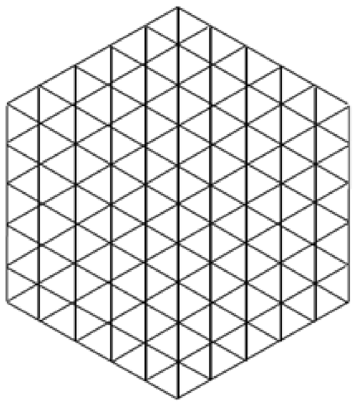

Figure 2: A 6-dimensional hexagonal network

Let H be the graph of hexagonal network HX_n. By calculation, we obtain that $|V(HX_n)| = 3n^2 - 3n + 1$ and $|E(HX_n)| = 9n^2 - 15n + 6$. From Figure 2, one can see that the vertices of HX_n are either of degree 3, 4 or 6. Then $\Delta(H) = 6$. In H, by algebraic method, there are five types of edges based on the degree of the end vertices of each edge as follows:

$$E_{34} = \{uv \in E(H) | d_H(u) = 3, d_H(v) = 4\}, \quad |E_{34}| = 12.$$

$$E_{36} = \{uv \in E(H) | d_H(u) = 3, d_H(v) = 6\}, \quad |E_{36}| = 6.$$

$$E_{44} = \{uv \in E(H) | d_H(u) = d_H(v) = 4\}, \quad |E_{44}| = 6n - 18.$$

$$E_{46} = \{uv \in E(H) | d_G(u) = 4, d_G(v) = 6\}, \quad |E_{46}| = 12n - 24.$$

$$E_{66} = \{uv \in E(H) | d_G(u) = d_G(v) = 6\}, \quad |E_{66}| = 9n^2 - 33n + 30.$$

Clearly, we have $c_u = \Delta(H) - d_H(u) + 1 = 7 - d_H(u)$. Thus there are five types of reverse edges as follows:

$$
\begin{aligned}
CE_{43} &= \{uv \in E(H) | c_u = 4, c_v = 3\}, \quad && |CE_{43}| = 12.\\
CE_{41} &= \{uv \in E(H) | c_u = 4, c_v = 1\}, \quad && |CE_{41}| = 6.\\
CE_{33} &= \{uv \in E(H) | c_u = c_v = 3\}, \quad && |CE_{33}| = 6n - 18.\\
CE_{31} &= \{uv \in E(H) | c_u = 3, c_v = 1\}, \quad && |CE_{31}| = 12n - 24.\\
CE_{11} &= \{uv \in E(H) | c_u = c_v = 1\}, \quad && |CE_{11}| = 9n^2 - 33n + 30.
\end{aligned}
$$

In the following theorem, we compute the geometric-arithmetic reverse index of hexagonal networks.

Theorem 3.1. *The geometric-arithmetic reverse index of hexgonal networks is given by*

$$GAC\left(HX_n\right) = 9n^2 + \left(6\sqrt{3} - 27\right)n + \left(\frac{48\sqrt{3}}{7} + \frac{24}{5} + 12\right).$$

Proof. By definition, we have

$$GAC\left(H\right) = \sum_{uv \in E(H)} \frac{2\sqrt{c_u c_v}}{c_u + c_v}.$$

Thus

$$
\begin{aligned}
GAC\left(HX_n\right) &= \sum_{CE_{43}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{41}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{33}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{31}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{11}} \frac{2\sqrt{c_u c_v}}{c_u + c_v}\\
&= \left(\frac{2\sqrt{4 \times 3}}{4+3}\right) 12 + \left(\frac{2\sqrt{4 \times 1}}{4+1}\right) 6 + \left(\frac{2\sqrt{3 \times 3}}{3+3}\right)(6n - 18)\\
&\quad + \left(\frac{2\sqrt{3 \times 1}}{3+1}\right)(12n - 24) + \left(\frac{2\sqrt{1 \times 1}}{1+1}\right)\left(9n^2 - 33n + 30\right)\\
&= 9n^2 + \left(6\sqrt{3} - 27\right)n + \left(\frac{48\sqrt{3}}{7} + \frac{24}{5} + 12\right).
\end{aligned}
$$

□

In the following theorem, we compute the sum connectivity reverse index of hexagonal networks.

Theorem 3.2. *The sum connectivity reverse index of hexgonal networks is given by*

$$SC\left(HX_n\right) = \frac{9}{\sqrt{2}}n^2 + \left(\sqrt{6} + 6 - \frac{33}{\sqrt{2}}\right)n + \left(\frac{12}{\sqrt{7}} + \frac{6}{\sqrt{5}} - 3\sqrt{6} - 12 + \frac{30}{\sqrt{2}}\right).$$

Proof. By definition, we have

$$SC\left(H\right) = \sum_{uv \in E(H)} \frac{1}{\sqrt{c_u + c_v}}.$$

Thus

$$
\begin{aligned}
SC\left(HX_n\right) &= \sum_{CE_{43}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{41}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{33}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{31}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{11}} \frac{1}{\sqrt{c_u + c_v}}\\
&= \left(\frac{1}{\sqrt{4+3}}\right) 12 + \left(\frac{1}{\sqrt{4+1}}\right) 6 + \left(\frac{1}{\sqrt{3+3}}\right)(6n - 18)\\
&\quad + \left(\frac{1}{\sqrt{3+1}}\right)(12n - 24) + \left(\frac{1}{\sqrt{1+1}}\right)\left(9n^2 - 33n + 30\right)\\
&= \frac{9}{\sqrt{2}}n^2 + \left(\sqrt{6} + 6 - \frac{33}{\sqrt{3}}\right)n + \left(\frac{12}{\sqrt{7}} + \frac{6}{\sqrt{5}} - 3\sqrt{6} - 12 + \frac{30}{\sqrt{2}}\right).
\end{aligned}
$$

□

References

[1] V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India, (2012).

[2] V.R.Kulli, *Atom bond connectivity reverse and product connectivity reverse indices of oxide and honeycomb networks*, submitted.

[3] V.R.Kulli, *On the sum connectivity reverse index of oxide and honeycomb networks*, Journal of Computer and Mathematical Sciences, 8(9)(2017), 408-413.

[4] S.Ediz, *Maximal graphs of the first reverse Zagreb beta index*, TWMS J. App. Eng. Math. accepted for publication.

[5] V.R.Kulli, *On the product connectivity reverse index of silicate and hexagonal networks*, International Journal of Mathematics and its Applications, 5(4B)(2017), 175179.

[6] V.R.Kulli, *Reverse Zagreb and reverse hyper-Zagreb indices and their polynomials of rhombus silicate networks*, submitted.

[7] I.Gutman and N.Trinajstic, *Graph theory and molecular orbitals, Total π-electron energy of alternant hydrocarbons*, Chem. Phys. Lett., 17(1972), 535-538.

[8] V.R.Kulli, *K-Banhatti indices of graphs*, Journal of Computer and Mathematical Sciences, 7(4)(2016), 213-218.

[9] V.R.Kulli, On K-hyper-Banhatti indices and coindices of graphs, International Research Journal of Pure Algebra, 6(5)(2016), 300-304.

[10] V.R.Kulli, *New K Banhatti topological indices*, International Journal of Fuzzy Mathematical Archive, 12(1)(2017), 29-37.

[11] V.R.Kulli, *The Gourava indices and coindices of graphs*, Annals of Pure and Applied Mathematics, 14(1)(2017), 33-38.

[12] V.R.Kulli, *Computation of general topological indices for titania nanotubes*, International Journal of Mathematical Archive, 7(12)(2016), 33-38.

[13] V.R.Kulli, *First multiplicative K Banhatti index and coindex of graphs*, Annals of Pure and Pure and Applied Mathematics, 11(2)(2016), 79-82.

[14] V.R.Kulli, *Some new multiplicative geometric-arithmetic indices*, Journal of Ultra Scientist of Physical Sciences A, 29(2)(2017), 52-57.

[15] V.R.Kulli, *General topological indices of tetrameric 1,3-adamantane*, International Journal of Current Research in Science and Technology, 3(8)(2017), 26-33.

[16] V.R.Kulli, *Computing topological indices of denrrimer nanostars*, International Journal of Mathematics and its Applications, 5(3-B)(2017), 163-169.

[17] I.Gutman, V.R.Kulli, B.Chaluvaraju and H.S.Baregowda, *On Banhatti and Zagreb indices*, Journal of the International Mathematical Virtual Institute, 7(2017), 53-67.

[18] V.R.Kulli, *Computation of some topological indices of certain networks*, International Journal of Mathematical Archive, 8(2)(2017), 99-106.