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Neighborhood Dakshayani Indices of Nanostructures

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Abstract: In this paper, we introduce the total neighborhood Dakshayani index, modified vertex neighborhood Dakshayani index, neighborhood Dakshayani inverse degree, neighborhood Dakshayani zeroth order index, F-neighborhood Dakshayani index and general first neighborhood Dakshayani index of a graph. Furthermore we propose the vertex neighborhood Dakshayani polynomial, total neighborhood Dakshayani polynomial and F-neighborhood Dakshayani polynomial of a graph. We determine exact formulas for line graphs of subdivision graphs of 2-D lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$.

MSC: 05C05, 05C12, 05C90.

Keywords: Neighborhood Dakshayani index, F-neighborhood Dakshayani index, general first neighborhood Dakshayani index, nanostructures.

1. Introduction

Let G be a finite simple, connected graph. The degree $d_G(v)$ of a vertex v is the number of edges incident to v. The edge connecting the vertices u and v will be denoted by uv. Let $N_G(v) = \{u : uv \in E(G)\}$. The closed neighborhood set of v is the set $N_G[v] = N_G(v) \cup \{v\}$. The set $N_G[v]$ is the set of closed neighborhood vertices of v. Let $D_G(v) = d_G(v) + \sum_{u \in N_G(v)} d_G(u)$ is the degree sum of closed neighborhood vertices of v. For other graph terminology and notation, refer [1]. Chemical Graph Theory is a branch of Mathematical Chemistry. A topological index is a numeric quantity from structural graph of a molecule. In Mathematical Chemistry, topological indices have found some applications especially in chemical documentation, isomer discrimination, QSAR/QSPR study [2, 3]. The line graph $L(G)$ of G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. The subdivision graph $S(G)$ of G is the graph obtained from G by replacing each of its edges by a path of length two. We need the following results.

Lemma 1.1. *Let G be a graph with p vertices and q edges. Then $S(G)$ has $p+q$ vertices and $2q$ edges.*

Lemma 1.2. *Let G be a (p,q) graph. Then $L(G)$ has q vertices and $\frac{1}{2}\sum_{i=1}^{p} d_G(u_i)^2 - q$ edges.*

Recently, the vertex neighborhood Dakshayani index was introduced by Kulli in [4], defined as

$$ND_v(G) = \sum_{u \in V(G)} D_G(u)^2.$$

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The total neighborhood Dakshayani index of a graph G is expressed as

$$T_D(G) = \sum_{u \in V(G)} D_G(u).$$

We now propose the following neighborhood Dakshayani indices. The modified vertex neighborhood Dakshayani index of G is defined as

$$^{m}ND_v(G) = \sum_{u \in V(G)} \frac{1}{D_G(u)^2}.$$

The neighborhood Dakshayani inverse degree of G is defined as

$$NDI(G) = \sum_{u \in V(G)} \frac{1}{D_G(u)}.$$

The neighborhood Dakshayani zeroth order index of G is defined as

$$NDZ(G) = \sum_{u \in V(G)} \frac{1}{\sqrt{D_G(u)}}.$$

In [5], Furtula et al. introduced a forgotten topological index or F-index, defined as

$$F(G) = \sum_{u \in V(G)} d_G(u)^3.$$

We introduce the F-neighborhood Dakshayani index of a graph G, defined as

$$FND(G) = \sum_{u \in V(G)} D_G(u)^3.$$

Recently, some different forgotten topological indices were studied, for example, in [6–10]. We continue this generalization and introduce the general first neighborhood Dakshayani index of a graph G, defined as

$$ND_v^a(G) = \sum_{u \in V(G)} D_G(u)^a \tag{1}$$

where a is a real number. We also propose the vertex neighborhood Dakshayani polynomial, total neighborhood Dakshayani polynomial, F-neighborhood Dakshayani polynomial of a graph, defined as

$$ND_1(G, x) = \sum_{u \in V(G)} x^{D_G(u)^2} \tag{2}$$

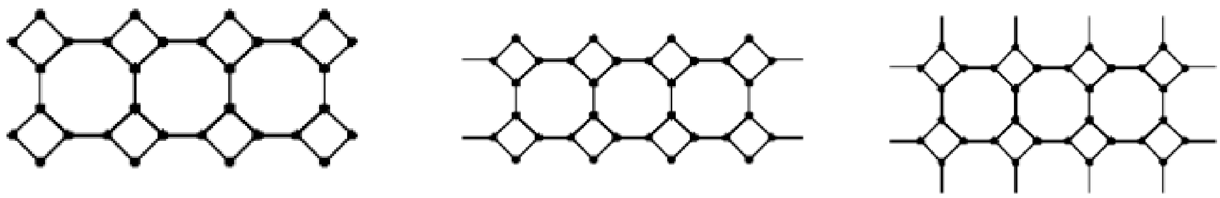
$$T_D(G, x) = \sum_{u \in V(G)} x^{D_G(u)} \tag{3}$$

$$FND(G, x) = \sum_{u \in V(G)} x^{D_G(u)^3} \tag{4}$$

Recently, some different polynomials were studied, for example, in [11–15]. In this paper, we determine explicit formulas for determining the vertex neighborhood Dakshayani index, modified vertex neighborhood Dakshayani index, the F-neighborhood Dakshayani polynomial, total neighborhood Dakshayani polynomial and F-neighborhood Dakshayani polynomial of line graphs of subdivision graphs of 2-D lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$. For more results on topological indices of line graphs of subdivision graphs see [16–20].

2. 2-D Lattice, Nanotube and Nanotorus of $TUC_4C_8[p, q]$

In this section, we consider the graph of 2-D lattice, nanotube, nanotorus of $TUC_4C_8[p, q]$. Let p denote the number of squares in a row and q denote the number of rows of squares in the graph of $TUC_4C_8[p, q]$. These graphs are shown in Figure 1.

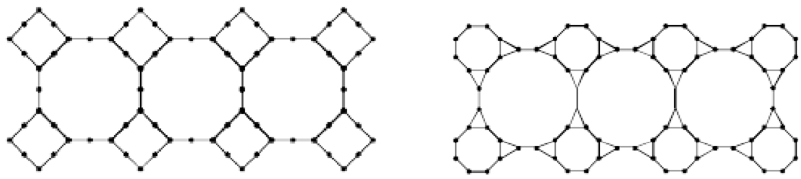


(a) 2D-lattice of $TUC_4C_8[4,2]$ (b) $TUC_4C_8[4,2]$ nanotube (c) $TUC_4C_8[4,2]$ nanotorus

Figure 1:

3. 2-D Lattice of $TUC_4C_8[p,q]$

Let G be the line graph of subdivision graph of $2D$-lattice of $TUC_4C_8[p,q]$. This graph is shown in Figure 2(b).



(a) Subdivision graph of 2-D lattice of $TUC_4C_8[4,2]$ (b) Line graph of subdivision graph of 2-D lattice of $TUC_4C_8[4,2]$

Figure 2:

A graph of 2-D lattice of $TUC_4C_8[p,q]$ has $4pq$ vertices and $6pq-p-q$ edges. By Lemma 1.1, The subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$ is a graph with $10pq-p-q$ vertices and $2(6pq-p-q)$ edges. Thus by Lemma 1.2, G has $2(6pq-p-q)$ vertices and $18pq-5-p-5q$ edges. The vertex partition based on the degree sum of closed neighborhood vertices of each vertex is obtained, as given in Table 1 and Table 2.

$D_G(u) \setminus u \in V(G)$	6	7	11	12
Number of vertices	8	$4(p+q-2)$	$4(p+q-2)$	$2(6pq-5p-5q+4)$

Table 1: Vertex partition of G with $p>1$, $q>1$

$D_G(u) \setminus u \in V(G)$	6	7	11	12
Number of vertices	8	$4(p-1)$	$4(p-1)$	$2(p-1)$

Table 2: Vertex partition of G with $p>1$, $q=1$

Theorem 3.1. *Let G be the line graph of subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$. Then the general vertex neighborhood Dakshayani index of G is*

$$ND_v^a(G) = 8\times 6^a + 4(p+q-2)(7^a+11^a) + 2(6pq-5p-5q+4)(12)^a, \quad \text{if } p>1,\ q>1, \tag{5}$$

$$= 8\times 6^a + (4\times 7^a + 4\times 11^a + 2\times 12^a)(p-1), \quad \text{if } p>1,\ q=1 \tag{6}$$

Proof.

Case 1: Suppose $p > 1$, $q > 1$. From equation (1) and by using Table 1, we have

$$\begin{aligned} ND_v^a(G) &= \sum_{u \in V(G)} D_G(u)^a \\ &= 8 \times 6^a + 4(p+q-2) \times 7^a + 4(p+q-2) \times 11^a + 2(6pq - 5p - 5q + 4) \times 12^a \\ &= 8 \times 6^a + 4(p+q-2)(7^a + 11^a) + 2(6pq - 5p - 5q) \times 12^a. \end{aligned}$$

Case 2: Suppose $p > 1$, $q = 1$. By using equation (1) and Table 2, we deduce

$$\begin{aligned} ND_v^a(G) &= \sum_{u \in V(G)} D_G(u)^a \\ &= 8 \times 6^a + 4(p-1) \times 7^a + 4(p-1) \times 11^a + 2(p-1) \times 12^a \\ &= 8 \times 6^a + (4 \times 7^a + 4 \times 11^a + 2 \times 12^a)(p-1). \end{aligned}$$

□

We obtain the following results by using Theorem 3.1.

Corollary 3.2 ([4])**.** *The vertex neighborhood Dakshayani index of G is*

$$\begin{aligned} ND_v(G) &= 1728pq - 760(p+q) + 80, && \text{if } p > 1,\ q > 1, \\ &= 968p - 680, && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = 2$ in equations (5) and (6), we get the desired results. □

Corollary 3.3. *The total neighborhood Dakshayani index of* G *is*

$$\begin{aligned} T_D(G) &= 144pq - 48(p+q), && \text{if } p > 1,\ q > 1, \\ &= 96p - 48, && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = 1$ in equations (5) and (6), we get the desired results. □

Corollary 3.4. *The modified vertex neighborhood Dakshayani index of* G *is*

$$\begin{aligned} {}^mND_v(G) &= \frac{8}{6^2} + \left(\frac{1}{7^2} + \frac{1}{11^2}\right) 4(p+q-2) + \frac{1}{12^2} 2(6pq - 5p - 5q + 4), && \text{if } p > 1,\ q > 1, \\ &= \frac{8}{6^2} + \left(\frac{4}{7^2} + \frac{4}{11^2} + \frac{2}{12^2}\right)(p-1), && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = -2$ in equations (5) and (6), we get the desired results. □

Corollary 3.5. *The neighborhood Dakshayani inverse degree of G is*

$$\begin{aligned} NDID(G) &= \frac{4}{3} + \frac{72}{77}(p+q-2) + \frac{1}{6}(6pq - 5p - 5q + 4), && \text{if } p > 1,\ q > 1, \\ &= \frac{509}{462}p - \frac{107}{462}, && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = -1$ in equations (5) and (6), we obtain the desired results. □

Corollary 3.6. *The neighborhood Dakshayani zeroth order index of G is*

$$NDZ(G) = \frac{8}{\sqrt{6}} + \left(\frac{1}{\sqrt{7}} + \frac{1}{\sqrt{11}}\right) 4(p+q-2) + \frac{1}{\sqrt{3}}(6pq-5p-5q+4), \quad \text{if } p>1,\ q>1,$$
$$= \frac{8}{\sqrt{6}} + \left(\frac{4}{\sqrt{7}} + \frac{4}{\sqrt{11}} + \frac{2}{\sqrt{12}}\right)(p-1), \quad \text{if } p>1,\ q=1.$$

Proof. Put $a = -\frac{1}{2}$ in equations (5) and (6), we get the desired results. □

Corollary 3.7. *The F-neighborhood Dakshayani index of G is*

$$FND(G) = 20736pq - 10584p + 2160, \quad \text{if } p>1,\ q>1,$$
$$= 10152p - 8424, \quad \text{if } p>1,\ q=1.$$

Proof. Put $a = 3$ in equations (5) and (6), we obtain the desired results. □

Theorem 3.8. *Let G be the line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$. Then the vertex neighborhood Dakshayani polynomial of G is*

$$ND_1(G,x) = 8x^{36} + 4(p+q-2)x^{49} + 4(p+q-2)x^{121} + 2(6pq-5p-5q+4)x^{144}, \quad \text{if } p>1,\ q>1,$$
$$= 8x^{36} + 4(p-1)x^{49} + 4(p-1)x^{121} + 2(p-1)x^{144}, \quad \text{if } p>1,\ q=1.$$

Proof.

Case 1: Suppose $p>1$ and $q>1$. From equation (2) and by using Table 1, we have

$$ND_1(G,x) = \sum_{u \in V(G)} x^{D_G(u)^2}$$
$$= 8 \times x^{6^2} + 4(p+q-2) \times x^{7^2} + 4(p+q-2) \times x^{11^2} + 2(6pq-5p-5q+4) \times x^{12^2},$$
$$= 8x^{36} + 4(p+q-2)x^{49} + 4(p+q-2)x^{121} + 2(6pq-5p-5q+4)x^{144},$$

Case 2: Suppose $p>1$ and $q=1$. By using equation (2) and Table 2, we obtain

$$ND_1(G,x) = \sum_{u \in V(G)} x^{D_G(u)^2}$$
$$= 8 \times x^{6^2} + 4(p-1)x^{7^2} + 4(p-1) \times x^{11^2} + 2(p-1) \times x^{12^2},$$
$$= 8x^{36} + 4(p-1)x^{49} + 4(p-1)x^{121} + 2(p-1)x^{144}.$$

□

Theorem 3.9. *Let G be the line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$. Then the total neighborhood Dakshayani polynomial of G is*

$$T_D(G,x) = 8x^6 + 4(p+q-2)x^9 + 4(p+q-2)x^{11} + 2(6pq-5p-5q+4)x^{12}, \quad \text{if } p>1,\ q>1,$$
$$= 8x^6 + 4(p-1)x^7 + 4(p-1)x^{11} + 2(p-1)x^{12}, \quad \text{if } p>1,\ q=1.$$

Proof.

Case 1: Suppose $p > 1$ and $q > 1$. From equation (3) and by using Table 1, we deduce

$$\begin{aligned} T_D(G,x) &= \sum_{u \in V(G)} x^{D_G(u)} \\ &= 8x^6 + 4(p+q-2)x^7 + 4(p+q-2)x^{11} + 2(6pq-5p-5q+4)x^{12}, \end{aligned}$$

Case 2: Suppose $p > 1$ and $q > 1$. By using equation (3) and Table 2, we derive

$$\begin{aligned} T_D(G,x) &= \sum_{u \in V(G)} x^{D_G(u)} \\ &= 8x^6 + 4(p-1)x^7 + 4(p-1)x^{11} + 2(p-1)x^{12}. \end{aligned}$$

□

Theorem 3.10. *Let G be the line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$. Then F-neighborhood Dakshayani Polynomial of G is*

$$\begin{aligned} FND(G,x) &= 8x^{216} + 4(p+q-2)x^{343} + 4(p+q-2)x^{1331} + 2(6pq-5p-5q+4)x^{1728}, \quad \text{if } p>1,\ q>1, \\ &= 8x^{216} + 4(p-1)x^{343} + 4(p-1)x^{1331} + 2(p-1)x^{1728}, \quad \text{if } p>1,\ q=1. \end{aligned}$$

Proof.

Case 1: Suppose $p > 1$ and $q > 1$. From equation (4) and using Table 1, we obtain

$$\begin{aligned} FND(G,x) &= \sum_{u \in V(G)} x^{D_G(u)^3} \\ &= 8 \times x^{6^3} + 4(p+q-2)x^{7^3} + 4(p+q-2) \times x^{11^3} + 2(6pq-5p-5q+4) \times x^{12^3}, \\ &= 8 \times x^{216} + 4(p+q-2)x^{343} + 4(p+q-2)x^{1331} + 2(pq-5p-5q+4)x^{1728}. \end{aligned}$$

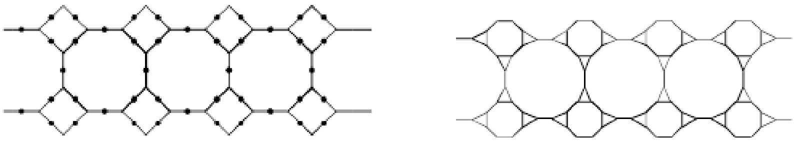
Case 2: Suppose $p > 1$ and $q = 1$. By using equation (4) and Table 2, we derive

$$\begin{aligned} FND(G,x) &= \sum_{u \in V(G)} x^{D_G(u)^3} \\ &= 8 \times x^{216} + 4(p-1)x^{343} + 4(p-1)x^{1331} + 2(p-1)x^{1728}. \end{aligned}$$

□

4. $TUC_4C_8[p,q]$ Nanotubes

Let H be the line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. This graph is shown in Figure 3(b). A graph of $TUC_4C_8[p,q]$ nanotube has $4pq$ vertices and $6pq - p$ edges. By Lemma 1.1, the subdivision graph of $TUC_4C_8[,q]$ nanotube has $10pq - p$ vertices and $12pq - 2p$ edges.



(a) Subdivision graph of $TUC_4C_8[4,2]$ nanotube

(b) Line graph of subdivision graph of $TUC_4C_8[4,2]$ nanotube

Figure 3:

Therefore from Lemma 1.2, H has 12 $pq-2p$ vertices and $18pq-5p$ edges. The vertex partition based on the degree sum of closed neighborhood vertices of each vertex is obtained as given in Table 3 and Table 4.

$D_H(u) \setminus u \in V(H)$	7	11	12
Number of vertices	$4p$	$4p$	$12pq-10p$

Table 3: Vertex partition of H if $p>1$, $q>1$

$D_H(u) \setminus u \in V(H)$	7	11	12
Number of vertices	$4p$	$4p$	$2p$

Table 4: Vertex partition of H if $p>1$, $q=1$

Theorem 4.1. *Let H be the line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then the general vertex neighborhood Dakshayani index of H is*

$$ND_v^a(H) = (7^a + 11^a)\,4p + 12^a\,(12pq - 10p)\,, \quad \text{if } p>1,\ q>1, \tag{7}$$

$$= (4 \times 7^a + 4 \times 11^a + 2 \times 12^a)\,p, \qquad \text{if } p>1,\ q=1 \tag{8}$$

Proof.

Case 1: Suppose $p>1$, $q>1$. From equation (1) and by using Table 3, we deduce

$$ND_v^a(H) = \sum_{u \in V(H)} D_H(u)^a$$

$$= 4p \times 7^a + 4p \times 11^a + (12pq - 10p)\,12^a$$

$$= (7^a + 11^a) + 12^a\,(12pq - 10p)\,.$$

Case 2: Suppose $p>1$, $q=1$. By using equation (2) and Table 4, we derive

$$ND_v^a(H) = \sum_{u \in V(H)} D_H(u)^a$$

$$= 4p \times 7^a + 4p \times 11^a + 2p \times 12^a$$

$$= (4 \times 7^a + 4 \times 11^a + 2 \times 12^a)\,p.$$

□

We establish the following results from Theorem 4.1.

Corollary 4.2 ([4]). *The vertex neighborhood Dakshayani index of H is*

$$\begin{aligned} ND_v(H) &= 1728pq - 760p, \quad \text{if } p > 1,\ q > 1, \\ &= 968p, \quad \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = 2$ in equations (7) and (8), we get the desired results. □

Corollary 4.3. *The total neighborhood Dakshayani index of H is*

$$\begin{aligned} T_D(H) &= 144pq - 48p, \quad \text{if } p > 1,\ q > 1, \\ &= 96p, \quad \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = 1$ in equations (7) and (8), we obtain the desired results. □

Corollary 4.4. *The modified vertex neighborhood Dakshayani index of H is*

$$\begin{aligned} {}^{m}ND_v(H) &= \frac{1}{12}pq + \left(\frac{4}{49} + \frac{4}{121} - \frac{10}{144}\right)p, \quad \text{if } p > 1,\ q > 1, \\ &= \left(\frac{4}{49} + \frac{4}{121} + \frac{2}{144}\right)p, \quad \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = -2$ in equations (7) and (8), we get the desired results. □

Corollary 4.5. *The neighborhood Dakshayani inverse degree of H is*

$$\begin{aligned} NDID(G) &= pq - \frac{47}{462}p, \quad \text{if } p > 1,\ q > 1, \\ &= \frac{1018}{924}p, \quad \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = -1$ in equations (7) and (8), we get the desired results. □

Corollary 4.6. *The neighborhood Dakshayani zeroth order index of H is*

$$\begin{aligned} NDZ(H) &= 2\sqrt{3}pq + \left(\frac{4}{\sqrt{7}} + \frac{4}{\sqrt{11}} - \frac{5}{\sqrt{3}}\right)p, \quad \text{if } p > 1,\ q = 1, \\ &= \left(\frac{4}{\sqrt{7}} + \frac{4}{\sqrt{11}} + \frac{1}{\sqrt{3}}\right)p, \quad \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = -\frac{1}{2}$ in equations (7) and (8), we obtain the desired results. □

Corollary 4.7. *The F neighborhood Dakshayani index of H is*

$$\begin{aligned} FND(H) &= 20736pq - 10584p, \quad \text{if } p > 1,\ q > 1, \\ &= 10152p, \quad \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = 3$ in equations (7) and (8), we get the desired results. □

Theorem 4.8. *Let H be the line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then the vertex neighborhood Dakshayani polynomial of H is*

$$\begin{aligned} ND_1(H,x) &= 4px^{49} + 4px^{121} + 4(12pq - 10p)x^{144}, && \text{if } p > 1,\ q > 1, \\ &= 4px^{49} + 4px^{121} + 2px^{144}, && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof.

Case 1: Suppose $p > 1$, $q > 1$. By using equation (2) and Table 3, we obtain

$$\begin{aligned} ND_1(H,x) &= \sum_{u \in V(H)} x^{D_H(u)^2} \\ &= 4px^{7^2} + 4px^{11^2} + (12pq - 10p)x^{12^2} \\ &= 4p^{49} + 4px^{121} + (12pq - 10p)x^{144}. \end{aligned}$$

Case 2: Suppose $p > 1$, $q = 1$. From equation (2) and Table 4, we have

$$\begin{aligned} ND_1(H,x) &= \sum_{u \in V(H)} x^{D_H(u)^2} \\ &= 4px^{7^2} + 4px^{11^2} + 2px^{12^2} \\ &= 4p^{49} + 4px^{121} + 2px^{144}. \end{aligned}$$

□

Theorem 4.9. *Let H be the line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then the total neighborhood Dakshayani polynomial of H is*

$$\begin{aligned} T_D(H,x) &= 4px^{7} + 4px^{11} + (12pq - 10p)x^{12}, && \text{if } p > 1,\ q > 1, \\ &= 4px^{7} + 4px^{11} + 2px^{12}, && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof.

Case 1: Suppose $p > 1$, $q > 1$. From equation (3) and by using Table 3, we have

$$\begin{aligned} T_D(H,x) &= \sum_{u \in V(H)} x^{D_H(u)} \\ &= 4px^{7} + 4px^{11} + (12pq - 10p)x^{12}. \end{aligned}$$

Case 2: Suppose $p > 1$, $q = 1$. By using equation (3) and Table 4, we obtain

$$\begin{aligned} T_D(H,x) &= \sum_{u \in V(H)} x^{D_H(u)} \\ &= 4px^{7} + 4px^{11} + 2px^{12}. \end{aligned}$$

□

Theorem 4.10. *Let H be the line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. The F-neighborhood Dakshayani polynomial of H is*

$$\begin{aligned} FND(H,x) &= 4px^{343} + 4px^{1331} + (12pq - 10p)x^{1728}, && \text{if } p > 1,\ q > 1, \\ &= 4px^{343} + 4px^{1331} + 2px^{1728}, && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof.

Case 1: Suppose $p > 1$, $q > 1$. By using equation (4) and Table 3, we deduce

$$FND(H,x) = \sum_{u \in V(H)} x^{D_H(u)^3}$$
$$= 4px^{7^3} + 4px^{11^3} + (12pq - 10p)\, x^{12^3}$$
$$= 4px^{343} + 4px^{1331} + (12pq - 10p)\, x^{1728}.$$

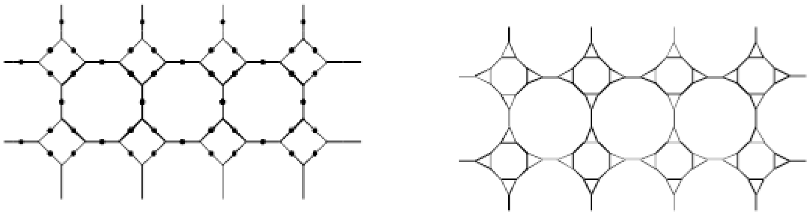
Case 2: Suppose $p > 1$ and $q = 1$. From equation (4) and by using Table 4, we derive

$$FND(H,x) = \sum_{u \in V(H)} x^{D_H(u)^3}$$
$$= 4px^{7^3} + 4px^{11^3} + 2px^{12^3}$$
$$= 4px^{343} + 4px^{1331} + 2px^{1728}.$$

□

5. $TUC_4C_8[p,q]$ Nanotorus

The line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotorus is presented in Figure 4(b).



(a) Subdivision graph of $TUC_4C_8[p,q]$ Nanotorus

(b) Line graph of subdivision graph of $TUC_4C_8[p,q]$ Nanotorus

Figure 4:

Theorem 5.1. *Let K be the line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotorus. Then*

(1). $ND_v^a(K) = 12pq \times 12^a$.

(2). $ND_v(K) = 1728pq$ [4]

(3). $T_D(K) = 144pq$.

(4). ${}^mND_v(K) = \frac{1}{12}pq$.

(5). $NDID(K) = pq$.

(6). $NDZ(K) = 2\sqrt{3}pq$.

(7). $FND(K) = 20736pq$.

(8). $ND_1(K,x) = 12pqx^{144}$.

(9). $T_D(K,x) = 12pqx^{12}$.

(10). $FND(K,x) = 12pqx^{1728}$.

Proof. A graph of $TUC_4C_8[p,q]$ nanotorus has $4pq$ vertices and $6pq$ edges. By Lemma 1.1, the subdivision graph of $TUC_4C_8[p,q]$ nanotorus is a graph with $10pq$ vertices and $12pq$ edges. Thus by Lemma 1.2, K has $12pq$ vertices and $18pq$ edges. Clearly degree of each vertex is 3. The vertex partition based on the degree sum of closed neighborhood vertices of each vertex is obtained as given in Table 5.

$D_K(u) \setminus u \in V(K)$	12
Number of vertices	$12pq$

Table 5: Vertex partition of K

From definitions and using Table 5, we obtain

$$ND_v^a(K) = \sum_{u\in V(K)} D_K(u)^a = 12pq \times 12^a.$$

$$ND_v(K) = \sum_{u\in V(K)} D_K(u)^2 = 12pq \times 12^2 = 1728pq.$$

$$T_D(K) = \sum_{u\in V(K)} D_K(u) = 12pq \times 12 = 144pq.$$

$${}^mND_v(K) = \sum_{u\in V(K)} \frac{1}{D_K(u)^2} = 12pq \times \frac{1}{12^2} = \frac{1}{12}pq.$$

$$NDID(K) = \sum_{u\in V(K)} \frac{1}{D_K(u)} = 12pq \times \frac{1}{12} = pq.$$

$$NDZ(K) = \sum_{u\in V(K)} \frac{1}{\sqrt{D_K(u)}} = 12pq \times \frac{1}{\sqrt{12}} = 2\sqrt{3}pq.$$

$$FND(K) = \sum_{u\in V(K)} D_K(u)^3 = 12pq \times 12^3 = 20736pq.$$

$$ND_1(K,x) = \sum_{u\in V(K)} x^{D_K(u)^2} = 12pq \times x^{12^2} = 12pqx^{144}.$$

$$T_D(K,x) = \sum_{u\in V(K)} x^{D_K(u)} = 12pqx^{12}.$$

$$FND(K,x) = \sum_{u\in V(K)} x^{D_K(u)^3} = 12pqx^{12^3} = 12pqx^{1728}.$$

□

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