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# Neighborhood Indices of Nanostructures

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**Abstract:** A topological index is a numerical parameter mathematically derived from the graph structure. In this study, we propose the modified first neighborhood index, neighborhood inverse degree, neighborhood zeroth order index, F-neighborhood index and general first neighborhood index of a graph. Also we introduce the first neighborhood polynomial, total neighborhood polynomial and F-neighborhood polynomial of a graph. Furthermore we compote exact formulas for line graphs of subdivision graphs of 2D-lattice, nanotube and nantorus of $TUC_4C_8[p,q]$.

**MSC:** 05C05, 05C012, 05C090.

**Keywords:** F-neighborhood index, general first neighborhood index, nanostructures.



## 1. Introduction

A molecular graph is graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of the Chemical Sciences. Several topological indices have found many applications, especially, in QSPR/QSAR study, see [1, 2]. Let $G$ be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex $v$ is the number of vertices adjacent to $v$. The edge connecting the vertices $u$ and $v$ will be denoted by $uv$. Let $N_G(v) = \{u : uv \in E(G)\}$. Let $S_G(v) = \sum_{u\in N_G(v)} d_G(u)$ be the degree sum of neighbor vertices. The line graph $L(G)$ of a graph $G$ is the graph whose vertex set corresponds to the edges of $G$ such that two vertices of $L(G)$ are adjacent if the corresponding edges of $G$ are adjacent. The subdivision graph $S(G)$ of $G$ is the graph obtained from $G$ by replacing each of its edges by a path of length two. For undefined term and notation, we refer the reader to [3]. We need the following results.

**Lemma 1.1.** *Let $G$ be a $(p,q)$ graph. Then $S(G)$ has $p+q$ vertices and $2q$ edges.*

**Lemma 1.2.** *Let $G$ be a $(p,q)$ graph. Then $L(G)$ has $q$ vertices and $\frac{1}{2}\sum_{i=1}^{p} d_G(u)^2 - q$ edges.*

In [4], Graovac introduced the fifth $M_1$ and $M_2$ Zagreb indices, defined as

$$M_1G_5(G) = \sum_{uv\in E(G)} [S_G(u) + S_G(v)], \quad M_2G_5(G) = \sum_{uv\in E(G)} S_G(u)S_G(v).$$

In [5], Kulli proposed the fifth hyper $M_1$ and $M_2$ Zagreb indices, defined as

$$HM_1G_5(G) = \sum_{uv\in E(G)} [S_G(u) + S_G(v)]^2, \quad HM_2G_5(G) = \sum_{uv\in E(G)} [S_G(u)S_G(v)]^2.$$

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Recently, the fifth multiplicative Zagreb indices [6], fifth multiplicative hyper Zagreb indices [7], fifth multiplicative connectivity indices [7] were introduced and studied. Recently, the first neighborhood Zagreb index was introduced and studied by Basavanagoud [8] and Mondal [9], defined as

$$NM_1(G) = \sum_{u \in V(G)} S_G(u)^2 .$$

The total neighborhood index of a graph $G$ is expressed as [8, 9]

$$T_n(G) = \sum_{u \in V(G)} S_G(u) .$$

We introduce the following the neighborhood Zagreb indices. The modified first neighborhood index of a graph $G$ is defined as

$$^{m}NM_1(G) = \sum_{u \in V(G)} \frac{1}{S_G(u)^2}.$$

The neighborhood inverse degree of a graph $G$ is defined as

$$NID(G) = \sum_{u \in V(G)} \frac{1}{S_G(u)}.$$

The neighborhood zeroth order index of a graph $G$ is defined as

$$NZ(G) = \sum_{u \in V(G)} \frac{1}{\sqrt{S_G(u)}}.$$

The $F$-neighborhood index of a graph $G$ is defined as

$$FN(G) = \sum_{u \in V(G)} S_G(u)^3 .$$

The general first neighborhood index of a graph $G$ is defined as

$$NM_1^a(G) = \sum_{u \in V(G)} S_G(u)^a \tag{1}$$

where $a$ is a real number. We also introduce the first neighborhood polynomial, total neighborhood polynomial, $F$-neighborhood polynomial of a graph, defined as

$$NM_1(G,x) = \sum_{u \in V(G)} x^{S_G(u)^2}. \tag{2}$$

$$T_n(G,x) = \sum_{u \in V(G)} x^{S_G(u)}. \tag{3}$$

$$FN(G,x) = \sum_{u \in V(G)} x^{S_G(u)^3}. \tag{4}$$

For a graph $G$, the modified version of neighborhood connectivity index and its polynomial are defined as

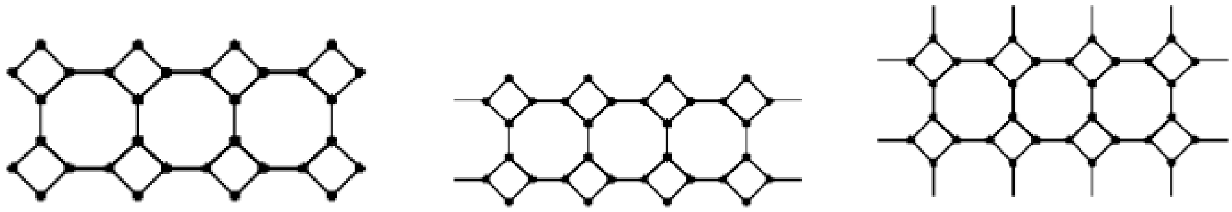
$$NC(G) = \sum_{u \in V(G)} d_G(u)\, S_G(u) , \tag{5}$$

$$NC(G,x) = \sum_{u \in V(G)} S_G(u)\, x^{d_G(u)}. \tag{6}$$

In this paper, we deduce explicit formulas for determining the modified first neighborhood index, neighborhood inverse degree, $F$-neighborhood index and general first neighborhood index of line graphs of subdivision graphs of 2-$D$ lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$. For more results on topological indices of line graphs of subdivision graphs see [10, 11, 12, 13, 14].

## 2. 2-D lattice, Nanotube and Nanotorus of $TUC_4C_8[p,q]$

In this section, we consider the graph of 2-D lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$, where $p$ is the number of squares in a row and $q$ is the number of rows of squares. These graphs are presented in Figure 1.

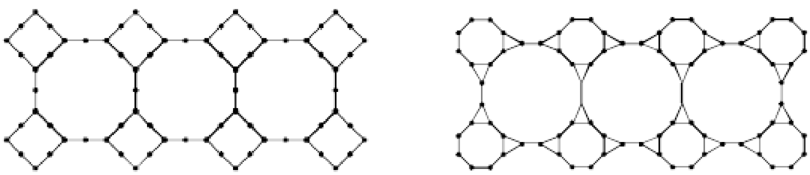


(a) 2D-lattice of $TUC_4C_8[4,2]$ (b) $TUC_4C_8[4,2]$ nanotube (c) $TUC_4C_8[4,2]$ nanotorus

Figure 1:

## 3. Results for 2-$D$ Lattice of $TUC_4C_8[p,q]$

The line graph of subdivision graph of 2-$D$ lattice of $TUC_4C_8[p,q]$ is shown in Figure 2(b).



(a) Subdivision graph of 2-$D$ lattice of $TUC_4C_8[4,2]$ (b) Line graph of subdivision graph of 2-$D$ lattice of $TUC_4C_8[4,2]$

Figure 2:

Let $G$ be a line graph of subdivision graph of 2-$D$ lattice of $TUC_4C_8[p,q]$. The 2-$D$ lattice of $TUC_4C_8[p,q]$ is a graph with $4pq$ vertices and $6pq-p-q$ edges. By Lemma 1.1, the subdivision graph of 2-$D$ lattice of $TUC_4C_8[p,q]$ is a graph with $10pq-p-q$ vertices and $2(6pq-p-q)$ edges, Thus by Lemma 1.2, $G$ has $2(6pq-p-q)$ vertices and $18pq-5p-5q$ edges. Clearly, the vertices of $G$ are either of degree 2 or 3, see Figure 2(b). The vertex partition based on the degree sum of neighbor vertices is obtained as given in Table 1 and Table 2.

| $S_G(u) \setminus u \in V(G)$ | 4 | 5 | 8 | 9 |
|---|---|---|---|---|
| Number of vertices | 8 | $4(p+q-2)$ | $4(p+q-2)$ | $2(6pq-5p-5q+4)$ |

Table 1: Vertex partition of $G$ when $p>1$, $q>1$

| $S_G(u) \setminus u \in V(G)$ | 4 | 5 | 8 | 9 |
|---|---|---|---|---|
| Number of vertices | 8 | $4(p-1)$ | $4(p-1)$ | $2(p-1)$ |

Table 2: Vertex partition of $G$ when $p>1$, $q=1$

**Theorem 3.1.** *Let $G$ be a line graph of subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$. Then the general first neighborhood index of $G$ is*

$$NM_1^a II\,(G) = 8\times 4^a + (5^a+8^a)\,4\,(p+q-2) + 2\,(6pq-5p-5q+4)\,9^a, \quad \text{if } p>1,\ q>1, \tag{7}$$

$$= 8\times 4^a + (4\times 5^a + 4\times 8^a + 2\times 9^a)\,(p-1)\,, \quad \text{if } p>1,\ q=1. \tag{8}$$

*Proof.* **Case 1:** Let $p>1$ and $q>1$.

From equation (1) and by using Table 1, we obtain

$$NM_1^a\,(G) = \sum_{u\in V(G)} S_G\,(u)^a$$
$$= 8\times 4^a + 4\,(p+q-2)\,5^a + 4\,(p+q-2)\,8^a + 2\,(6pq-5p-5q+4)\,9^a$$

Thus, $NM_1^a\,(G) = 8\times 4^a + 4\,(p+q-2)\,(5^a+8^a) + 2\,(6pq-5p-5q+4)\,9^a$.

**Case 2:** Let $p>1$ and $q=1$.

From equation (1) and by using Table 2, we obtain

$$NM_1^a\,(G) = \sum_{u\in V(G)} S_G\,(u)^a$$
$$= 8\times 4^a + 4\,(p-1)\,5^a + 4\,(p-1)\,8^a + 2\,(p-1)\,9^a$$

Thus, $NM_1^a\,(G) = 8\times 4^a + (4\times 5^a + 4\times 8^a + 2\times 9^a)\,(p-1)$. ☐

We establish the following results by using Theorem 3.1.

**Corollary 3.2** ([8])**.** *The first neighborhood Zagreb index of $G$ is*

$$NM_1(G) = 972pq - 454(p+q) + 64, \quad \text{when } p>1,\ q>1,$$
$$= 518p - 390, \quad \text{when } p>1,\ q=1.$$

*Proof.* Put $a=2$ in equations (7) and (8), we get the desired results. ☐

**Corollary 3.3.** *The total neighborhood index of $G$ is*

$$T_n(G) = 108pq - 38(p+q) + 72, \quad \text{if } p>1,\ q>1,$$
$$= 70p - 38, \quad \text{if } p>1,\ q=1.$$

*Proof.* Put $a=1$ in equations (7) and (8), we get the desired results. ☐

**Corollary 3.4.** *The modified first neighborhood index of $G$ is*

$${}^{m}NM_1\,(G) = \frac{2}{81}\,(6pq-5p-5q+4) + \left(\frac{1}{25}+\frac{1}{64}\right)4\,(p+q-2) + \frac{1}{2}, \quad \text{if } p>1,\ q>1,$$
$$= \left(\frac{4}{25}+\frac{4}{64}+\frac{2}{81}\right)p + \frac{1}{2} - \left(\frac{4}{25}+\frac{4}{64}+\frac{2}{81}\right), \quad \text{if } p>1,\ q=1.$$

*Proof.* Put $a=-2$ in equations (7) and (8), we get desired results. ☐

**Corollary 3.5.** *The neighborhood inverse degree of* $G$ *is*

$$NID(G) = \frac{2}{9}(6pq - 5p - 5q + 4) + \frac{52}{40} + (p + q - 2) + 2, \quad \text{if } p > 1,\ q > 1,$$
$$= \frac{141}{90}p + \frac{13}{30}, \quad \text{if } p > 1,\ q = 1.$$

*Proof.* Put $a = -1$ in equations (7) and (8), we obtain the desired results. ☐

**Corollary 3.6.** *The neighborhood zeroth order index of* $G$ *is*

$$NZ(G) = \frac{2}{3}(6pq - 5p - 5q + 4) + \left(\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{8}}\right) 4(p + q - 2) + 4, \quad \text{if } p > 1,\ q > 1,$$
$$= \left(\frac{4}{\sqrt{5}} + \frac{4}{\sqrt{8}} + \frac{2}{3}\right) p + 4 - \left(\frac{4}{\sqrt{5}} + \frac{4}{\sqrt{8}} + \frac{2}{3}\right), \quad \text{if } p > 1,\ q = 1.$$

*Proof.* Put $a = -\frac{1}{2}$ in equations (7) and (8), we get the desired results. ☐

**Corollary 3.7.** *The F-neighborhood index of* $G$ *is*

$$FN(G) = 8748pq - 4742(p + q) + 1248, \quad \text{when } p > 1,\ q > 1,$$
$$= 4006p - 3494, \quad \text{when } p > 1,\ q = 1.$$

*Proof.* Put $a = 3$ in equations (7) and (8), we obtain the desired results. ☐

**Theorem 3.8.** *Let* $G$ *be a line graph of subdivision graph of 2D-lattice of* $TUC_4C_8[p, q]$*. Then*

$$NM_1(G, x) = 8 \times x^{16} + 4(p + q - 2)x^{25} + 4(p + q - 2)x^{64} + 2(6pq - 5p - 5q + 4)x^{81}, \quad \text{if } p > 1,\ q > 1,$$
$$= 8 \times x^{16} + 4(p - 1)x^{25} + 4(p - 1)x^{64} + 2(p - 1)x^{81}, \quad \text{if } p > 1,\ q = 1.$$

*Proof.* **Case 1:** Let $p > 1$ and $q > 1$.

From equation (2) and by using Table 1, we have

$$NM_1(G, x) = \sum_{u \in V(G)} x^{S_G(u)^2}$$
$$= 8 \times x^{16} + 4(p + q - 2)x^{25} + 4(p + q - 2)x^{64} + 2(6pq - 5p - 5q + 4)x^{81}.$$

**Case 2:** Let $p > 1$ and $q = 1$.

From equation (2) and by using Table 2, we obtain

$$NM_1(G, x) = \sum_{u \in V(G)} x^{S_G(u)^2}$$
$$= 8 \times x^{16} \times 4(p - 1)x^{25} + 4(p - 1)x^{64} + 2(p - 1)x^{81}.$$

☐

**Theorem 3.9.** *Let* $G$ *be a line graph of subdivision graph of 2D-lattice of* $TUC_4C_8[p, q]$*. Then*

$$T_n(G, x) = 8 \times x^4 + 4(p + q - 2)x^5 + 4(p + q - 2)x^8 + 2(6pq - 5p - 5q + 4)x^9, \quad \text{if } p > 1,\ q > 1,$$
$$= 8 \times x^4 + 4(p - 1)x^5 + 4(p - 1)x^8 + 2(p - 1)x^9, \quad \text{if } p > 1,\ q = 1.$$

*Proof.* **Case 1:** Suppose $p > 1$and $q > 1$.

From equation (3) and by using Table 1, we deduce

$$\begin{aligned} T_n(G,x) &= \sum_{u \in V(G)} x^{S_G(u)} \\ &= 8 \times x^4 + 4(p+q-2)x^5 + 4(p+q-2)x^8 + 2(6pq-5p-5q+4)x^9. \end{aligned}$$

**Case 2:** Suppose $p > 1$ and $q = 1$.

By using equation (3) and using Table 2, we derive

$$\begin{aligned} T_n(G,x) &= \sum_{u \in V(G)} x^{S_G(u)} \\ &= 8 \times x^4 + 4(p-1)x^5 + 4(p-1)x^8 + 2(p-1)x^9. \end{aligned}$$

□

**Theorem 3.10.** *Let $G$ be a line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$. Then*

$$\begin{aligned} FN(G,x) &= 8x^{64} + 4(p+q-2)x^{125} + 4(p+q-2)x^{512} + 2(6pq-5p-5q+4)x^{729}, && \text{if } p>1,\ q>1, \\ &= 8x^{64} + 4(p-1)x^{125} + 4(p-1)x^{512} + 2(p-1)x^{729}, && \text{if } p>1,\ q=1. \end{aligned}$$

*Proof.* **Case 1:** Suppose $p > 1$, $q > 1$.

By using equation (4) and Table 1, we have

$$\begin{aligned} FN(G,x) &= \sum_{u \in V(G)} x^{S_G(u)^3} \\ &= 8x^{4^3} + 4(p+q-2)x^{5^3} + 4(p+q-2)x^{8^3} + 2(6pq-5p-5q+4)x^{9^3}. \\ &= 8x^{64} + 4(p+q-2)x^{125} + 4(p+q-2)x^{512} + 2(6pq-5p-5q+4)x^{729}. \end{aligned}$$

**Case 2:** Suppose $p > 1$, $q = 1$.

From equation (4) and by using Table 2, we obtain

$$\begin{aligned} FN(G,x) &= \sum_{u \in V(G)} x^{S_G(u)^3} \\ &= 8 \times x^{64} + 4(p-1)x^{125} + 4(p-1)x^{512} + 2(p-1)x^{729}. \end{aligned}$$

□

**Theorem 3.11.** *Let $G$ be a line graph of subdivision graph 2D-lattice of $TUC_4C_8[p,q]$. Then the modified version of neighborhood connectivity index and its polynomial of $G$ are given by*

$$\begin{aligned} NC(G) &= 324pq - 134(p+q) + 8, && \text{if } p>1,\ q>1, \\ &= 190p - 126, && \text{if } p>1,\ q=1. \\ NC(G,x) &= [20(p+q)-8]x^2 + [108pq - 58(p+q) + 8]x^3, && \text{if } p>1,\ q>1, \\ &= (20p-12)x^2 + (42p-42)x^3, && \text{if } p>1,\ q=1. \end{aligned}$$

*Proof.* Consider the graph $G$ which is shown in Figure 2(b). The vertex partitions of $G$ are given in Table 3 and Table 4.

| $d_G(u)$, $S_G(v)$ | (2, 4) | (2, 5) | (3, 8) | (3, 9) |
|---|---|---|---|---|
| Number of vertices | 8 | $4(p+q-2)$ | $4(p+q-2)$ | $2(6pq-5p-5q+4)$ |

Table 3: Vertex partition of $G$ when $p>1$, $q>1$

| $d_G(u)$, $S_G(v)$ | (2, 4) | (2, 5) | (3,8) | (3, 9) |
|---|---|---|---|---|
| Number of vertices | 8 | $4(p-1)$ | $4(p-1)$ | $2(p-1)$ |

Table 4: Vertex partition of $G$ when $p>1$, $q=1$

**Case 1:** Suppose $p>1$, $q>1$.

(i). By using equation (5) and Table 3, we derive

$$\begin{aligned} NC(G) &= \sum_{u\in V(G)} d_G(u)\, S_G(u) \\ &= (2\times 4)\,8 + (2\times 5)\,4\,(p+q-2) + (3\times 8)4\,(p+q-2) + (3\times 9)\,2\,(6pq-5p-5q+4) \\ &= 324pq - 134\,(p+q) + 8 \end{aligned}$$

(ii). From equation (6) and using Table 3, we deduce

$$\begin{aligned} NC(G,x) &= \sum_{u\in V(G)} S_G(u)\, x^{d_G(u)} \\ &= 8\times 4x^2 + 4\,(p+q-2)\,5x^2 + 4\,(p+q-2)\,8x^3 + 2\,(6pq-5p-5q+4)\,9x^3 \\ &= [20\,(p+q)-8]\,x^2 + [108pq - 58\,(p+q)+8]\,x^3. \end{aligned}$$

**Case 2:** Suppose $p>1$, $q=1$.

(i). From equation (5) and by using Table 4, we have

$$\begin{aligned} NC(G) &= \sum_{u\in V(G)} d_G(u)\, S_G(u) \\ &= (2\times 4)\,8 + (2\times 5)\,4\,(p-1) + (3\times 8)\,4\,(p-1) + (3\times 9)\,2\,(p-1) \\ &= 190p - 126. \end{aligned}$$

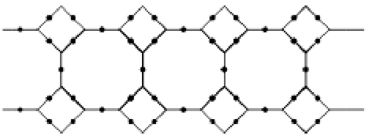
(ii). From equation (6) an by using Table 4, we obtain

$$\begin{aligned} NC(G,x) &= \sum_{u\in V(G)} S_G(u)\, x^{d_G(u)} \\ &= 8\times 4x^2 + 4\,(p-1)\,5x^2 + 4\,(p-1)\,8x^3 + 2\,(p-1)\,9x^3 \\ &= (20p-12)\,x^2 + (42p-42)\,x^3. \end{aligned}$$

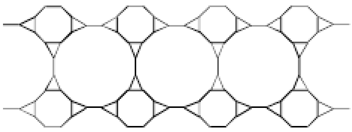
□

# 4. Results for $TUC_4C_8[p,q]$ Nanotube

The line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube is shown in Figure 3 (b).



(a) Subdivision graph of $TUC_4C_8[4,2]$ nanotube



(b) Line graph of subdivision graph of $TUC_4C_8[4,2]$ nanotube

Figure 3:

Let $H$ be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. The graph of $TUC_4C_8[p,q]$ nanotube has $4pq$ vertices and $6pq-p$ edges. By Lemma 2.1, the subdivision graph of $TUC_4C_8[p,q]$ nanotube is a graph with $10pq-p$ vertices and $12pq-2p$ edges. Thus by Lemma 1.2, $H$ has $12pq-2p$ vertices and $18pq-5p$ edges. Clearly the vertices of $H$ are either of degree 2 or 3. The vertex partition based on the degree sum of neighbor vertices of each vertex is given in Table 5 and Table 6.

| $S_H(u) \setminus u \in V(H)$ | 5 | 8 | 9 |
|---|---|---|---|
| Number of vertices | $4p$ | $4p$ | $12pq-10p$ |

Table 5: Vertex partition of $H$ if $p>1,\ q>1$

| $S_H(u) \setminus u \in V(H)$ | 5 | 8 | 9 |
|---|---|---|---|
| Number of vertices | $4p$ | $4p$ | $2p$ |

Table 6: Vertex partition of $H$ if $p>1,\ q=1$

**Theorem 4.1.** *Let $H$ be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then the general first neighborhood index of $H$ is*

$$NM_1^a(H) = (5^a+8^a)\,4p + 9^a\,(12pq-10p)\,, \quad \textit{if } p>1,\ q>1, \tag{9}$$

$$= (5^a+8^a)\,4p + 9^a \times 2p, \quad \textit{if } p>1,\ q=1. \tag{10}$$

*Proof.* **Case 1:** Suppose $p>1$ and $q>1$.

From equation (1) and by using Table 5, we deduce

$$NM_1^a(H) = \sum_{u\in V(H)} S_H(u)^a$$

$$= 4p \times 5^a \times 4p \times 8^a + (12pq-10p) \times 9^a$$

$$= (5^a+8^a)\,4p + 9^a\,(12pq-10p)\,.$$

**Case 2:** Suppose $p>1$ and $q=1$.

By using equation (1) and Table 6, we derive

$$\begin{aligned} NM_1^a(H) &= \sum_{u \in V(H)} S_H(u)^a \\ &= 4p \times 5^a \times 4p \times 8^a + 2p \times 9^a \\ &= (5^a + 8^a)\,4p + 9^a \times 2p. \end{aligned}$$

□

We obtain the following results by Theorem 4.1.

**Corollary 4.2** ([8]). . *The first neighborhood Zagreb index of $H$ is*

$$\begin{aligned} NM_1(H) &= 972pq - 454p, && \text{if } p > 1,\ q > 1, \\ &= 518p, && \text{if } p > 1,\ q = 1. \end{aligned}$$

*Proof.* Put $a = 2$ in equations (9) and (10), we get the desired results. □

**Corollary 4.3.** *The total neighborhood index of $H$ is*

$$\begin{aligned} T_n(H) &= 108pq - 38p, && \text{if } p > 1,\ q > 1, \\ &= 70p, && \text{if } p > 1,\ q = 1. \end{aligned}$$

*Proof.* Put $a = 1$ in equations (9) and (10), we get the desired results. □

**Corollary 4.4.** *The modified first neighborhood index of $H$ is*

$$\begin{aligned} {}^mNM_1(H) &= \frac{4}{27}pq + \left(\frac{4}{25} + \frac{4}{64} - \frac{10}{81}\right)p, && \text{if } p > 1,\ q > 1, \\ &= \left(\frac{4}{25} + \frac{4}{64} + \frac{2}{81}\right)p, && \text{if } p > 1,\ q = 1. \end{aligned}$$

*Proof.* Put $a = -2$ in equations (9) and (10), we get the desired results. □

**Corollary 4.5.** *The neighborhood inverse degree of $H$ is*

$$\begin{aligned} NID(H) &= \frac{4}{3}pq + \frac{17}{90}p, && \text{if } p > 1,\ q > 1, \\ &= \frac{137}{90}p, && \text{if } p > 1,\ q = 1. \end{aligned}$$

*Proof.* Put $a = -1$ in equations (9) and (10), we get the desired results. □

**Corollary 4.6.** *The neighborhood zeroth order index of $H$ is*

$$\begin{aligned} NZ(H) &= 4pq + \left(\frac{4}{\sqrt{5}} + \frac{4}{\sqrt{8}} - \frac{10}{3}\right)p, && \text{if } p > 1,\ q > 1, \\ &= \left(\frac{4}{\sqrt{5}} + \frac{4}{\sqrt{8}} + \frac{2}{3}\right)p, && \text{if } p > 1,\ q = 1. \end{aligned}$$

*Proof.* Put $a = -\frac{1}{2}$ in equations (9) and (10), we get the desired results. □

**Corollary 4.7.** *The F-neighborhood index of $H$ is*

$$FN(H) = 8748pq - 4742p, \quad \text{if } p > 1,\ q > 1,$$
$$= 4006p, \qquad \text{if } p > 1,\ q = 1.$$

*Proof.* Put $a = 3$ in equations (9) and (10), we get the desired results. □

**Theorem 4.8.** *Let $H$ be a line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotube. Then*

$$NM_1(H, x) = 4px^{25} + 4px^{64} + (12pq - 10p)\, x^{81}, \quad \text{if } p > 1,\ q > 1,$$
$$= 4px^{25} + 4px^{64} + 2px^{81}, \qquad \text{if } p > 1,\ q = 1.$$

*Proof.* **Case 1:** Suppose $p > 1$ and $q > 1$.

By using equation (2) and Table 5, we obtain

$$NM_1(H, x) = \sum_{u \in V(H)} x^{S_H(u)^2}$$
$$= 4px^{5^2} \times 4px^{8^2} + (12pq - 10p)\, x^{9^2}$$
$$= 4px^{25} + 4px^{64} + (12pq - 10p)x^{81}.$$

**Case 2:** Suppose $p > 1$, $q = 1$.

From equation (2) and by using Table 6, we obtain

$$NM_1(H, x) = \sum_{u \in V(H)} x^{S_H(u)^2}$$
$$= 4px^{5^2} + 4px^{8^2} + 2px^{9^2}$$
$$= 4px^{25} + 4px^{64} + 2px^{81}.$$

□

**Theorem 4.9.** *Let $H$ be a line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotube. Then*

$$T_n(H, x) = 4px^5 + 4px^8 + (12pq - 10p)\, x^9, \quad \text{if } p > 1,\ q > 1,$$
$$= 4px^5 + 4px^8 + 2px^9, \qquad \text{if } p > 1,\ q = 1.$$

*Proof.* **Case 1:** Suppose $p > 1$, $q > 1$.

From equation (3) and by using Table 5, we deduce

$$T_n(H, x) = \sum_{u \in V(H)} x^{S_H(u)}$$
$$= 4px^5 + 4px^8 + (12pq - 10p)\, x^9.$$

**Case 2:** Suppose $p > 1$, $q = 1$.

By using equation (3) and Table 6, we derive

$$T_n(H, x) = \sum_{u \in V(H)} x^{S_H(u)}$$
$$= 4px^5 + 4px^8 + 2px^9.$$

□

**Theorem 4.10.** *Let $H$ be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then*

$$FN\left(H,x\right) = 4px^{125} + 4px^{512} + \left(12pq - 10p\right)x^{729}, \quad \text{if } p > 1,\ q > 1,$$
$$= 4px^{125} + 4px^{512} + 2px^{729}, \quad \text{if } p > 1,\ q = 1.$$

*Proof.* **Case 1:** Suppose $p > 1,\ q > 1$.

By using equation (4) and Table 5, we have

$$FN\left(H,x\right) = \sum_{u \in V(H)} x^{S_H(u)^3}$$
$$= 4px^{5^3} + 4px^{8^3} + \left(12pq - 10p\right)x^{9^3}$$
$$= 4px^{125} + 4px^{512} + \left(12pq - 10p\right)x^{729}.$$

**Case 2:** Suppose $p > 1,\ q = 1$.

From equation (4) and by using Table 6, we obtain

$$FN\left(H,x\right) = \sum_{u \in V(H)} x^{S_H(u)^3}$$
$$= 4px^{5^3} + 4px^{8^3} + 2px^{9^3}$$
$$= 4px^{125} + 4px^{512} + 2px^{729}.$$

□

**Theorem 4.11.** *Let $H$ be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then the modified version of neighborhood connectivity index and its polynomial of $H$ are given by*

$$NC\left(H\right) = 324pq - 134p, \quad \text{if } p > 1,\ q > 1,$$
$$= 190p, \quad \text{if } p > 1,\ q = 1.$$
$$NC\left(H,x\right) = 20px^2 + \left(108pq - 58p\right)x^3, \quad \text{if } p > 1,\ q > 1,$$
$$= 20px^2 + 50px^3, \quad \text{if } p > 1,\ q = 1.$$

*Proof.* Consider the graph $H$ which is shown in Figure 3(b). The vertex partitions of $H$ are given in Table 7 and Table 8.

| $d_H(u),\ S_H(u)$ | (2, 5) | (3, 8) | (3,9) |
|---|---|---|---|
| Number of vertices | $4p$ | $4p$ | $12pq - 10p$ |

Table 7: Vertex partition of $H$ when $p > 1,\ q > 1$

| $d_H(u),\ S_H(u)$ | (2, 5) | (3, 8) | (3,9) |
|---|---|---|---|
| Number of vertices | $4p$ | $4p$ | $2p$ |

Table 8: Vertex partition of $H$ when $p > 1,\ q = 1$

**Case 1:** Suppose $p > 1,\ q > 1$.

(i). From equation (5) and by using Table 7, we obtain

$$\begin{aligned} NC(H) &= \sum_{u\in V(H)} d_H(u) S_H(u) \\ &= (2\times 5)\,4p + (3\times 8)\,4p + (3\times 9)\,(12pq - 10p) \\ &= 324pq - 134p. \end{aligned}$$

(ii). From equation (6) and using Table 7, we have

$$\begin{aligned} NC(H,x) &= \sum_{u\in V(H)} S_H(u)\, x^{d_H(u)} \\ &= 4p\times 5x^2 + 4p\times 8x^3 + (12pq - 10p)\times 9x^3 \\ &= 20px^2 + (108pq - 58p)\,x^3. \end{aligned}$$

**Case 2:** Suppose $p > 1$, $q = 1$.

(i). By using equation (5) and Table 8, we deduce

$$\begin{aligned} NC(H) &= \sum_{u\in V(H)} d_H(u) S_H(u) \\ &= (2\times 5)\,4p + (3\times 8)\,4p + (3\times 9)\,2p \\ &= 190p. \end{aligned}$$

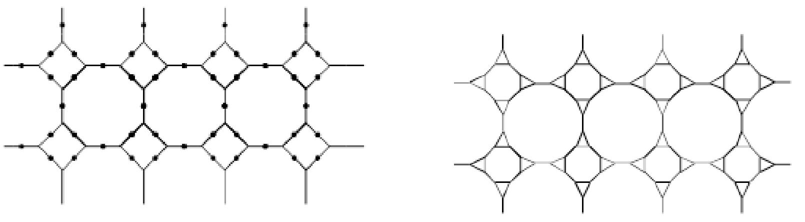
(ii). By using equation (6) and Table 8, we derive

$$\begin{aligned} NC(H,x) &= \sum_{u\in V(H)} S_H(u)\, x^{d_H(u)} \\ &= 4p\times 4x^2 + 4p\times 8x^3 + 2p\times 9x^3 \\ &= 20px^2 + 50px^3. \end{aligned}$$

□

## 5. Results for $TUC_4C_8[p,q]$ Nanotorus

The line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotorus is shown in Figure 4(b).



(a) Subdivision graph of $TUC_4C_8[p,q]$ nanotorus

(b) Line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotorus

Figure 4:

Let $K$ be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotorus. A graph of $TUC_4C_8[p,q]$ nanotorus has $4pq$ vertices and $6pq$ edges. By Lemma 1, the subdivision graph of $TUC_4C_8[p,q]$ nanotorus is a graph with $10pq$ vertices and $12pq$ edges. Thus by Lemma 1.2, $K$ has $12pq$ vertices and $18pq$ edges. Clearly the degree of each vertex is 3. The vertex partition based on the degree sum of neighbor vertices of each vertex is as given in Table 9.

| $d_K(u)$, $S_K(u) \setminus u \in V(K)$ | (3, 9) |
|---|---|
| Number of vertices | $12pq$ |

Table 9: Vertex partition of $K$

**Theorem 5.1.** *Let $K$ be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotorus. Then*

*(1).* $NM_1^a(K) = 9^a \times 12pq$.

*(2).* $NM_1(K) = 972pq$ *[8]*.

*(3).* $T_n(K) = 108pq$.

*(4).* ${}^mNM_1(K) = \frac{4}{27}pq$.

*(5).* $NID(K) = \frac{4}{3}pq$.

*(6).* $NZ(K) = 4pq$.

*(7).* $FN(K) = 729pq$.

*(8).* $NM_1(K,x) = 12pqx^{81}$.

*(9).* $T_n(K,x) = 12pqx^9$.

*(10).* $FN(K,x) = 12pqx^{729}$.

*(11).* $NC(K) = 324pq$.

*(12).* $NC(K,x) = 108pqx^3$.

*Proof.* By using definitions and Table 9, we obtain the desired results. □

## References


[1] I. Gutman and O.E. Polansky, *Mathematical Concepts in Organic Chemistry*, Springer, Berlin, (1986).

[2] V.R. Kulli, *Multiplicative Connectivity Indices of Nanostructures*, LAP LEMBERT Academic Publishing, (2018).

[3] V.R. Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India, (2012).

[4] A. Graovac, M. Ghorbani and M. A. Hosseinzadeh, *Computing fifth geometric-arithmetic index of nanostar dendrimers*, Journal of Mathematical Nanoscience, 1(1)(2011), 33-42.

[5] V. R. Kulli, *General fifth M-Zagreb indices and fifth M-Zagreb polynomials of PAMAM dendrimers*, International Journal of Fuzzy Mathematical Archive, 13(1)(2017), 99-103.

[6] V. R. Kulli, *Some new multiplicative geometric-arithmetic indices*, Journal of Ultra Scientist of Physical Sciences A, 29(2)(2017), 52-57.

[7] V. R. Kulli, *Some new fifth multiplicative Zagreb indices of PAMAM dendrimers*, Journal of Global Research in Mathematics, 5(2)(2018), 82-86.

[8] B. Basavanagoud, A. P. Barangi and S. M. Hosamani, *First neighbourhood Zagreb index of some nanostructures*, Proceedings IAM, 7(2)(2018), 178-193.

[9] S. Mondal. N. De and A. Pal, *On neighborhood index of product of graphs*, ArXiv: 1805.05273vi [*Math. Co*] 14 May 2018.

[10] A. Ashrafi and S. Yousefi, *Computing Wiener index of* $TUC_4C_8(S)$ *nanotorus*, MATCH Commun. Math. Comput. Chem., 57(2)(2017), 403-310.

[11] V. R. Kulli, *Computing fifth arithmetic-geometric index of certain nanostructures*, Journal of Computer and Mathematical Sciences, 8(7)(2017), 197-201.

[12] V. R. Kulli, *Two new multiplicative atom bond connectivity indices*, Annals of Pure and Applied Mathematics, 13(1)(2017), 1-7.

[13] M. F. Nadeem, S. Zafar and Z. Zahid, *On certain topological indices of the line graph of subdivision graphs*, Appl. Math. Comput., 271(2015), 790-794.

[14] G. Su and L. Xu, *Topological indices of the line graph of subdivision graph and their Schur-bounds*, Appl. Math. Comput., 253(2015), 395-401.